Lecture 30: Sorting Networks

COSC 273: Parallel and Distributed Computing

Spring 2023

Annoucements

Submission links today:

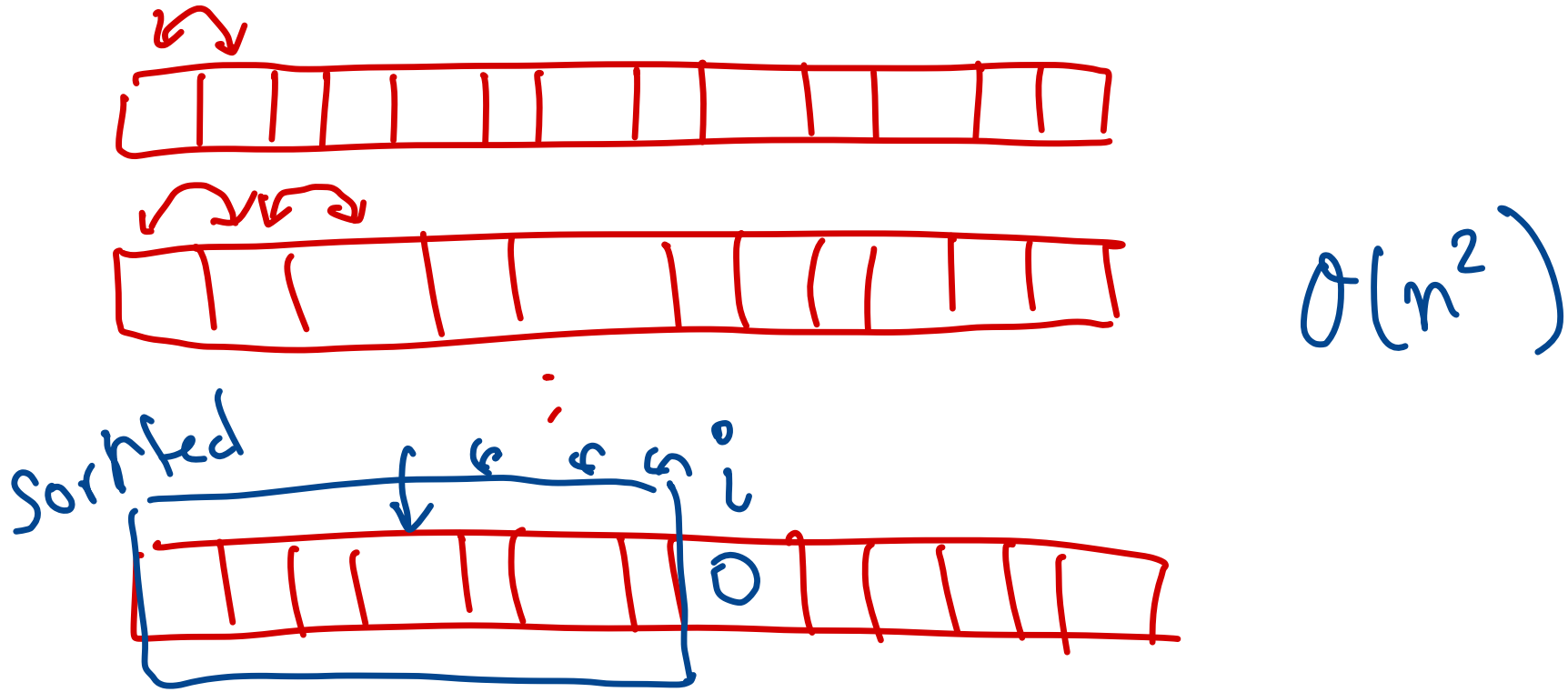
1. Final Project Leaderboard Submissions
2. Choose Your Own Adventure Baselines

Today

Sorting *small* arrays quickly

Insertion Sort, Revisited

```
for (int i = 1; i < data.length; ++i) {
    for (int j = i; j > 0; --j) {
        if (data[j-1] > data[j]) {
            swap(data, j-1, j)
        }
    }
}
```



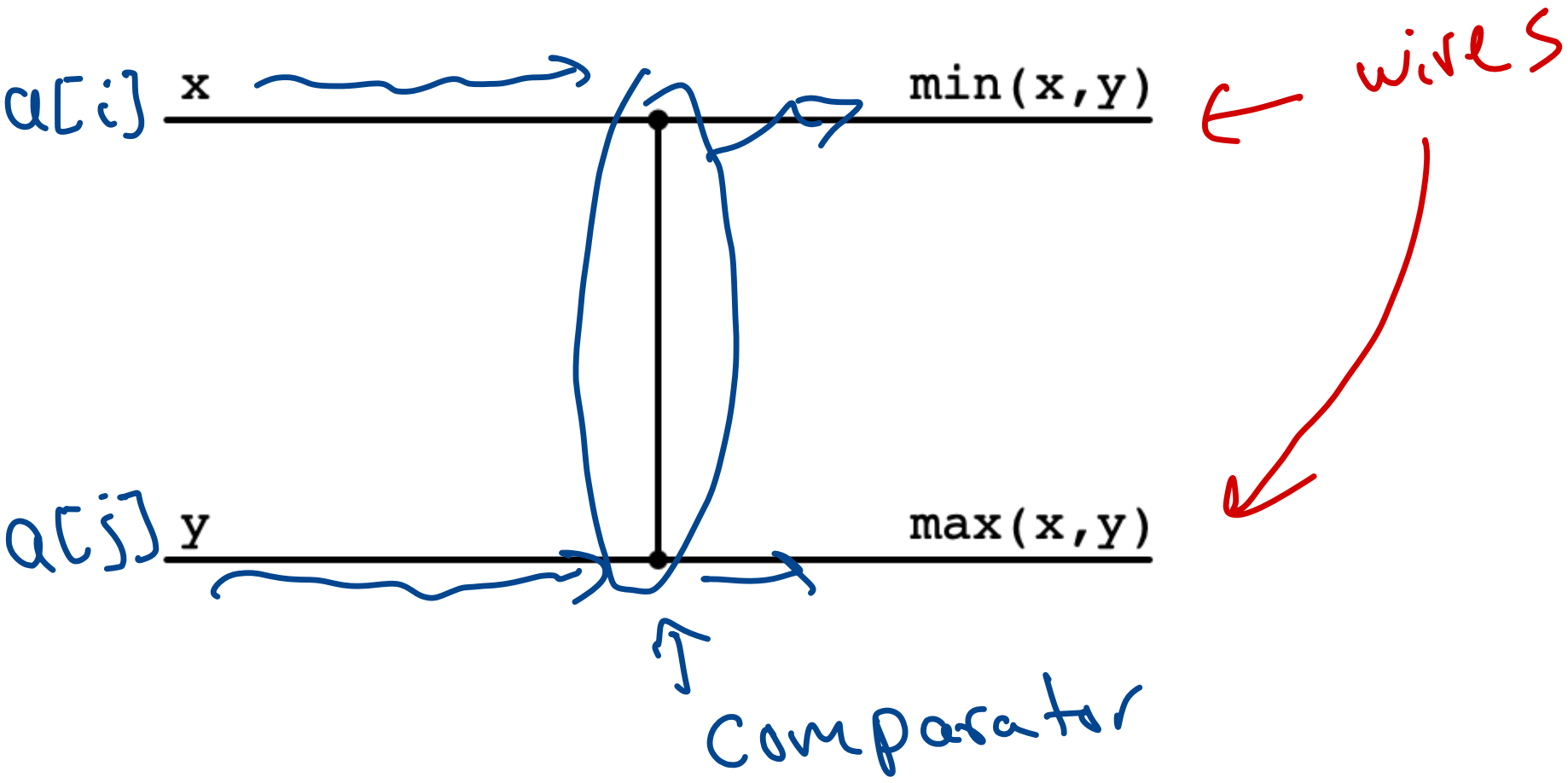
Appealing Features of Insertion Sort

```
for (int i = 1; i < data.length; ++i) {
    for (int j = i; j > 0; --j) {
        if (data[j-1] > data[j]) {
            swap(data, j-1, j)
        }
    }
}
```

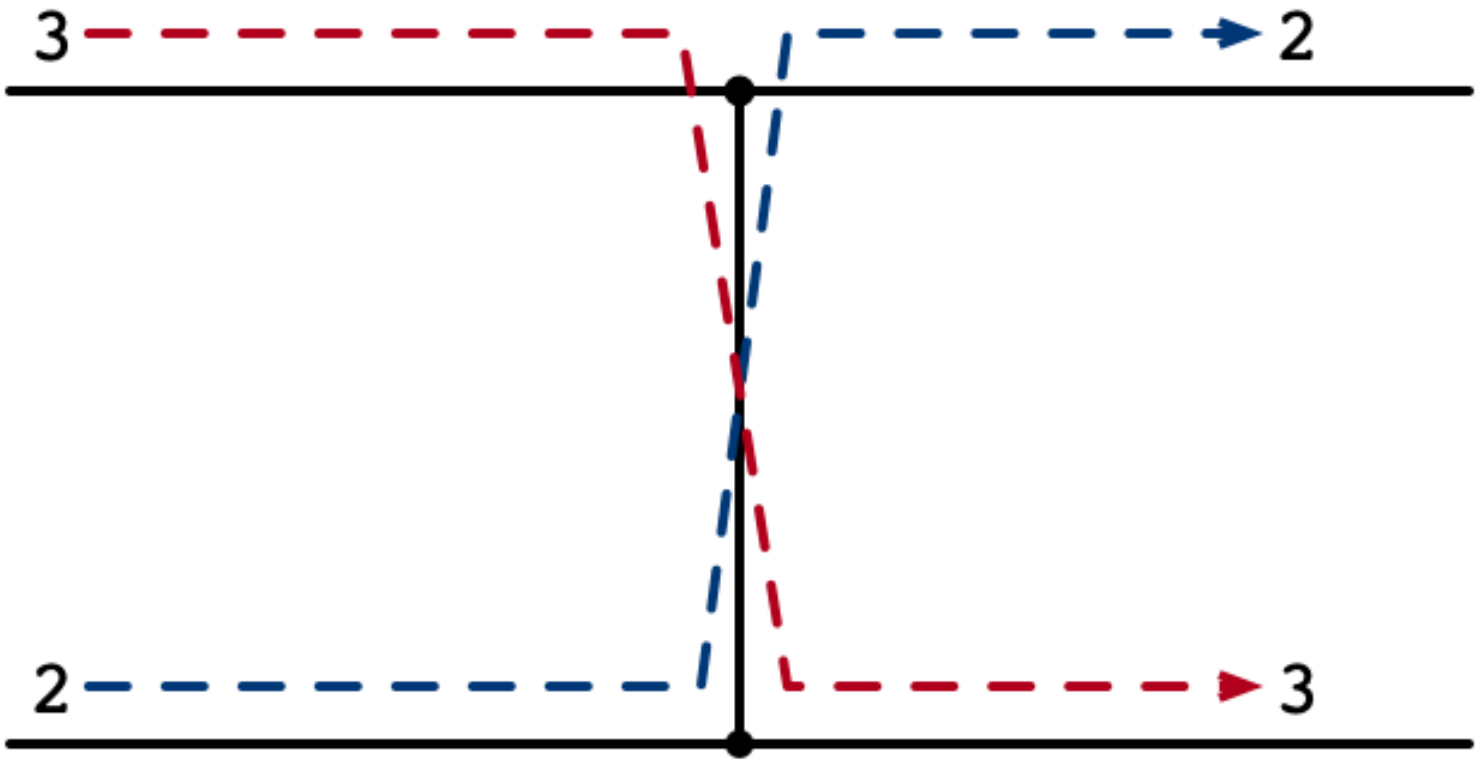
1. Only modifications are (adjacent) swaps
   - sorting is *in place*
2. Access pattern is independent of input
   - inputs always read/compared in same order
   - only difference between execution is outcomes of swaps

Comparators: Visualizing Swaps

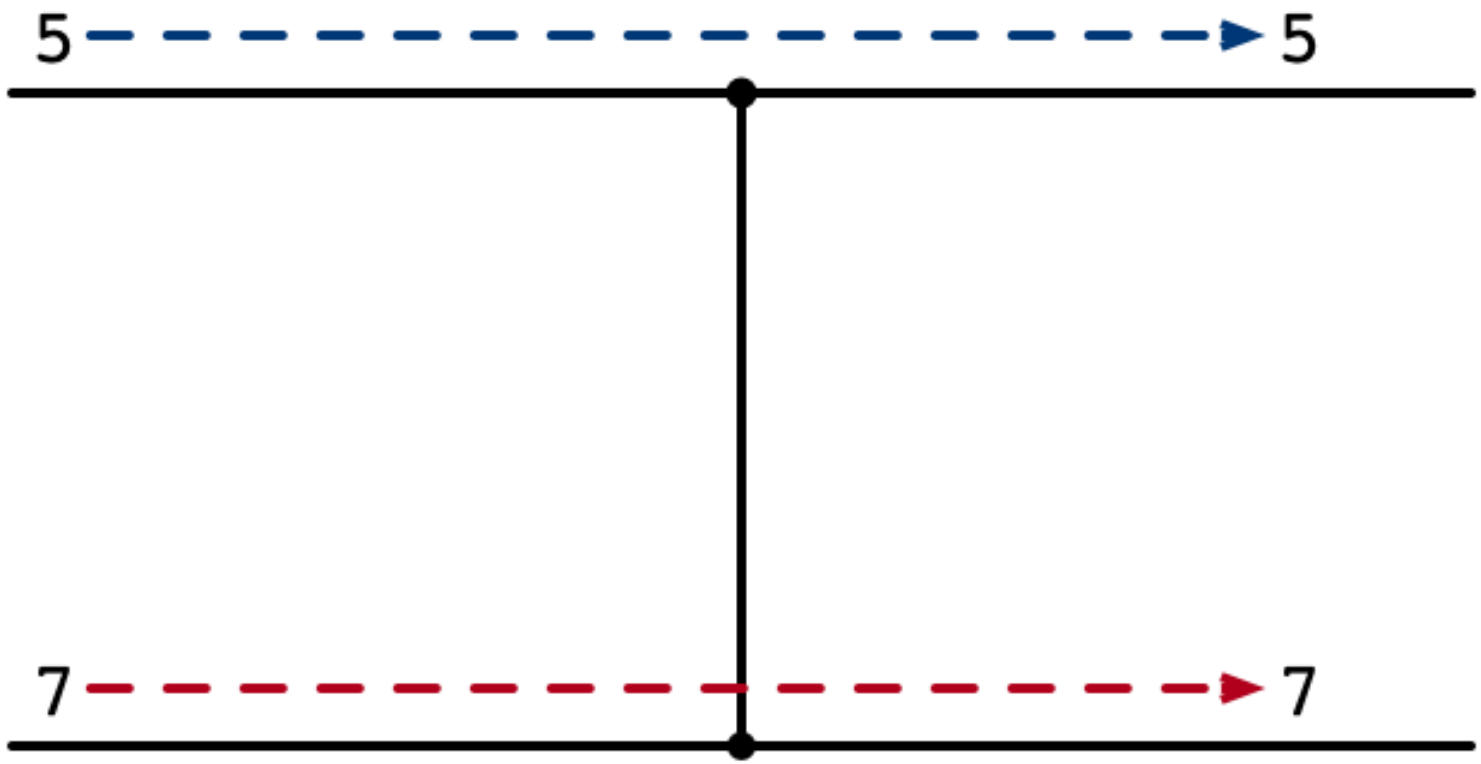
```
if (data[i] > data[j]) {
    swap(data, i, j)
}
```



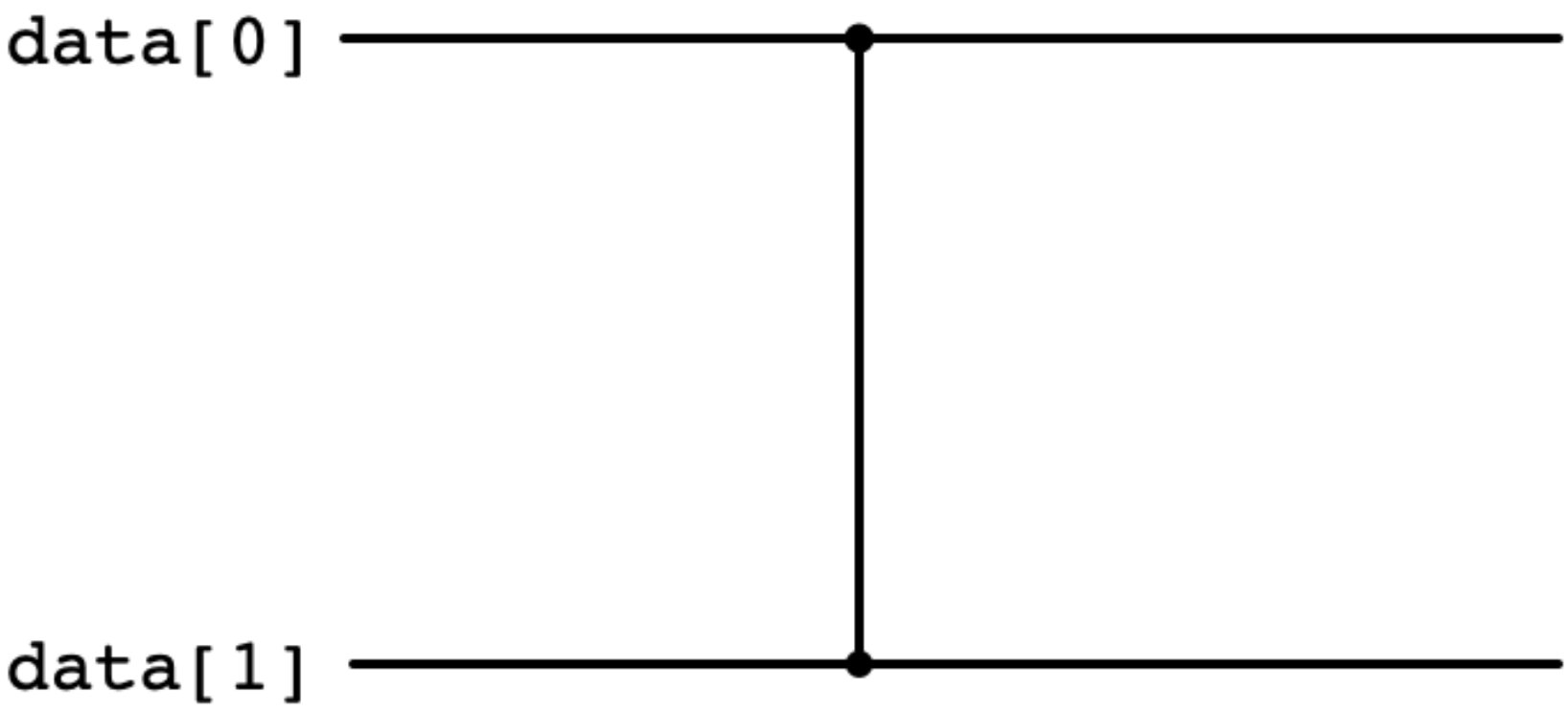
Comparator Swap



Comparator No Swap



Sorting Array of Two Elements



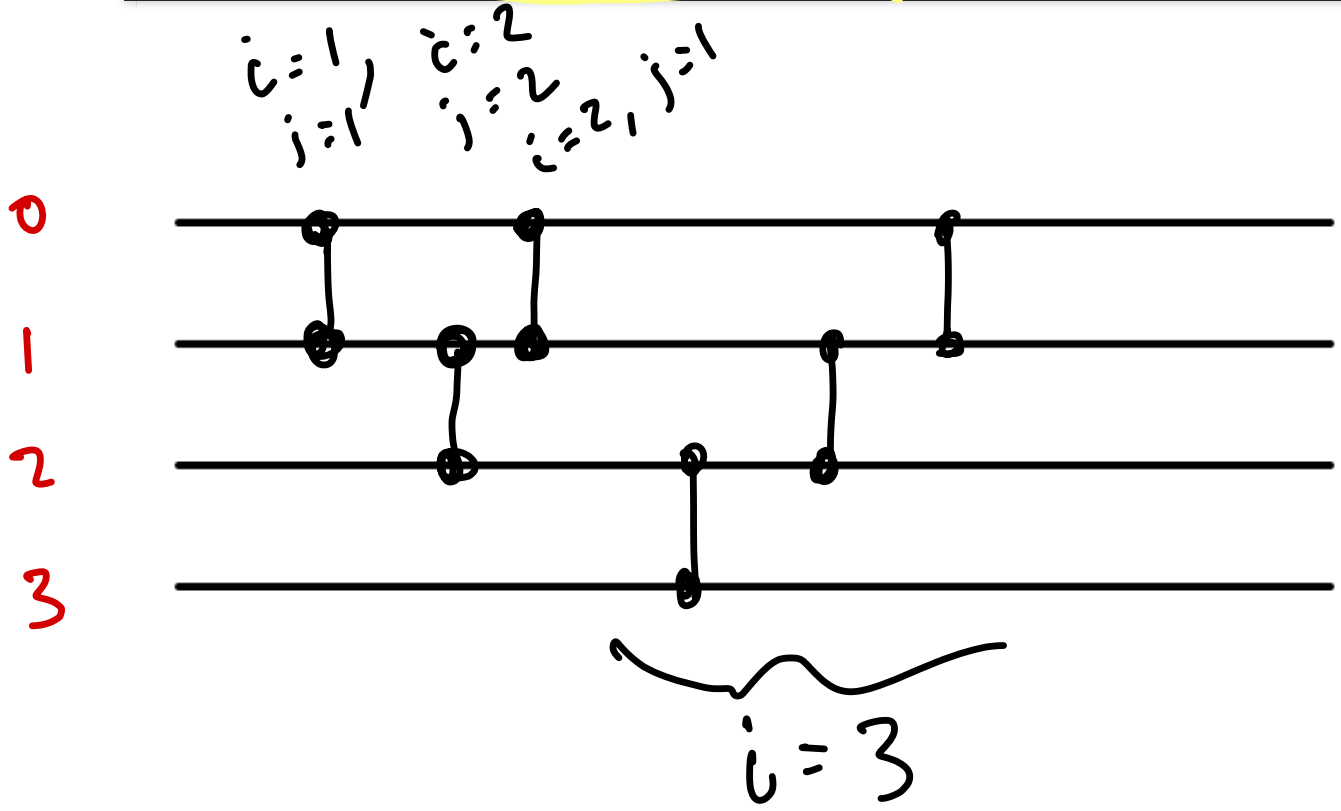
equiv to:

```
if (data[0] > data[1])
    swap(data, 0, 1)
```

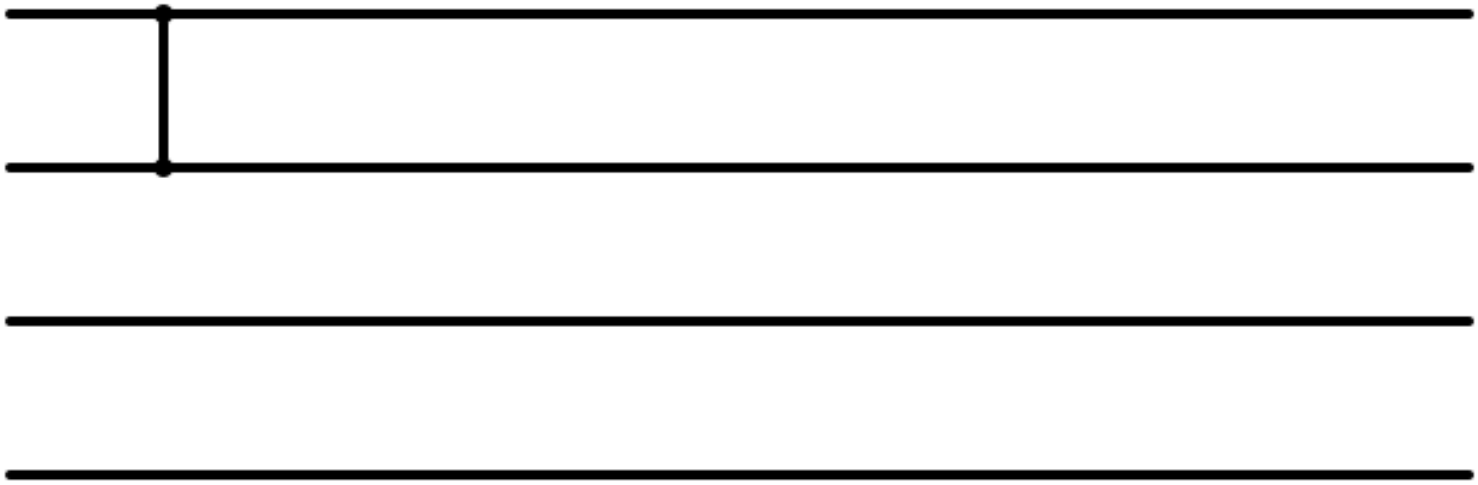
Insertion Sort

```
for (int i = 1; i < data.length; ++i) {
    for (int j = i; j > 0; --j) {
        if (data[j-1] > data[j]) {swap(data, j-1, j)}}}
```

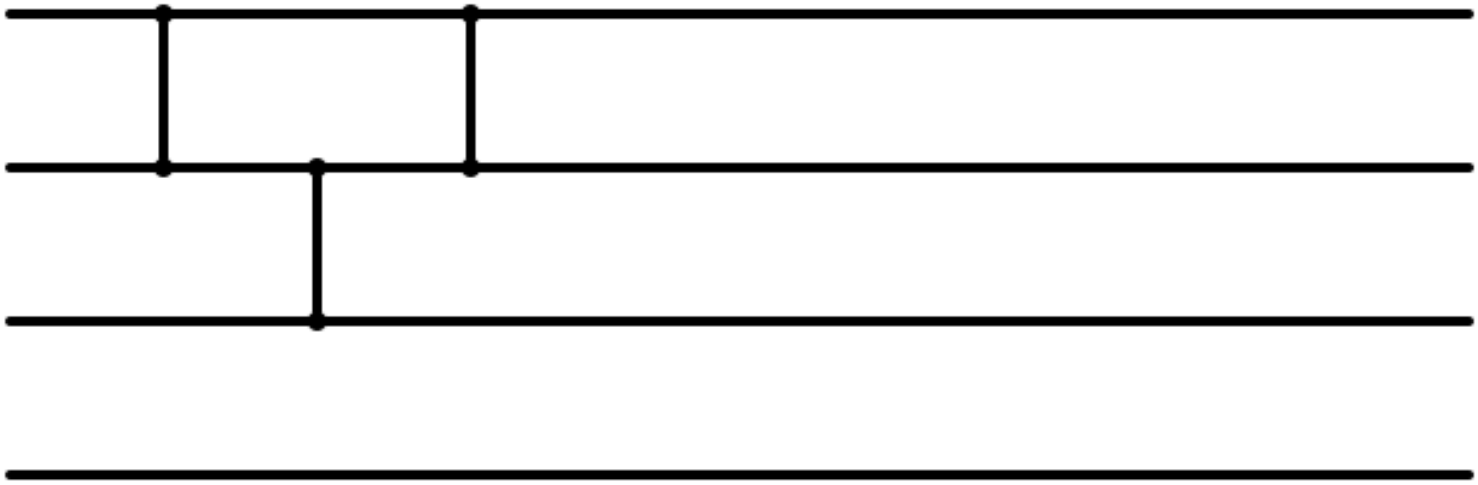
Comparator op



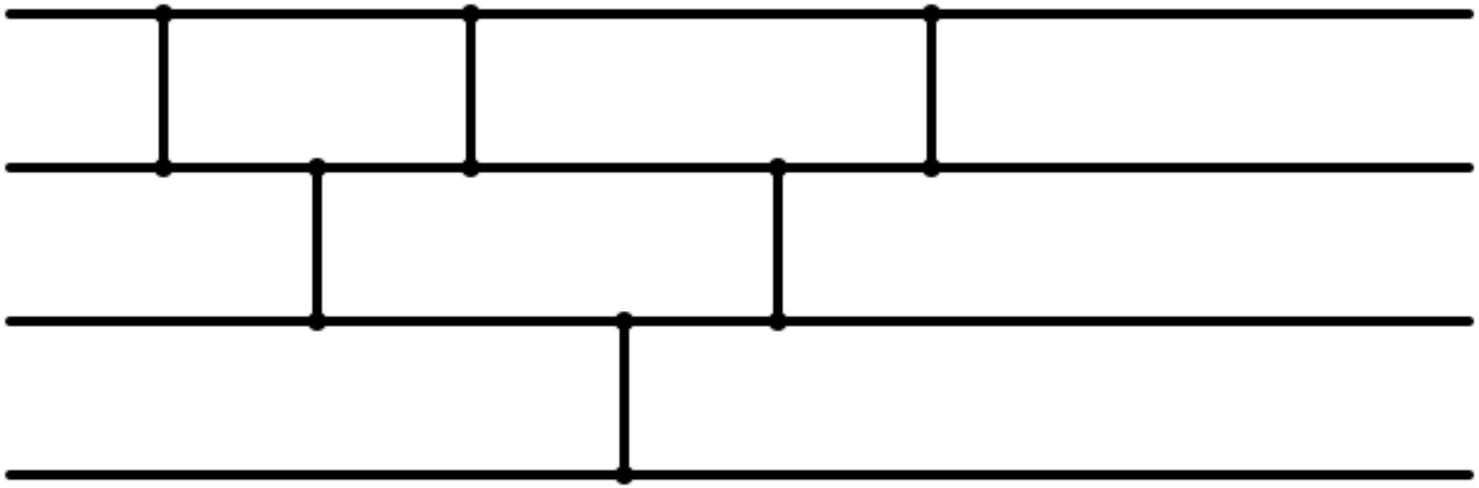
Insertion Sort: `i = 1`



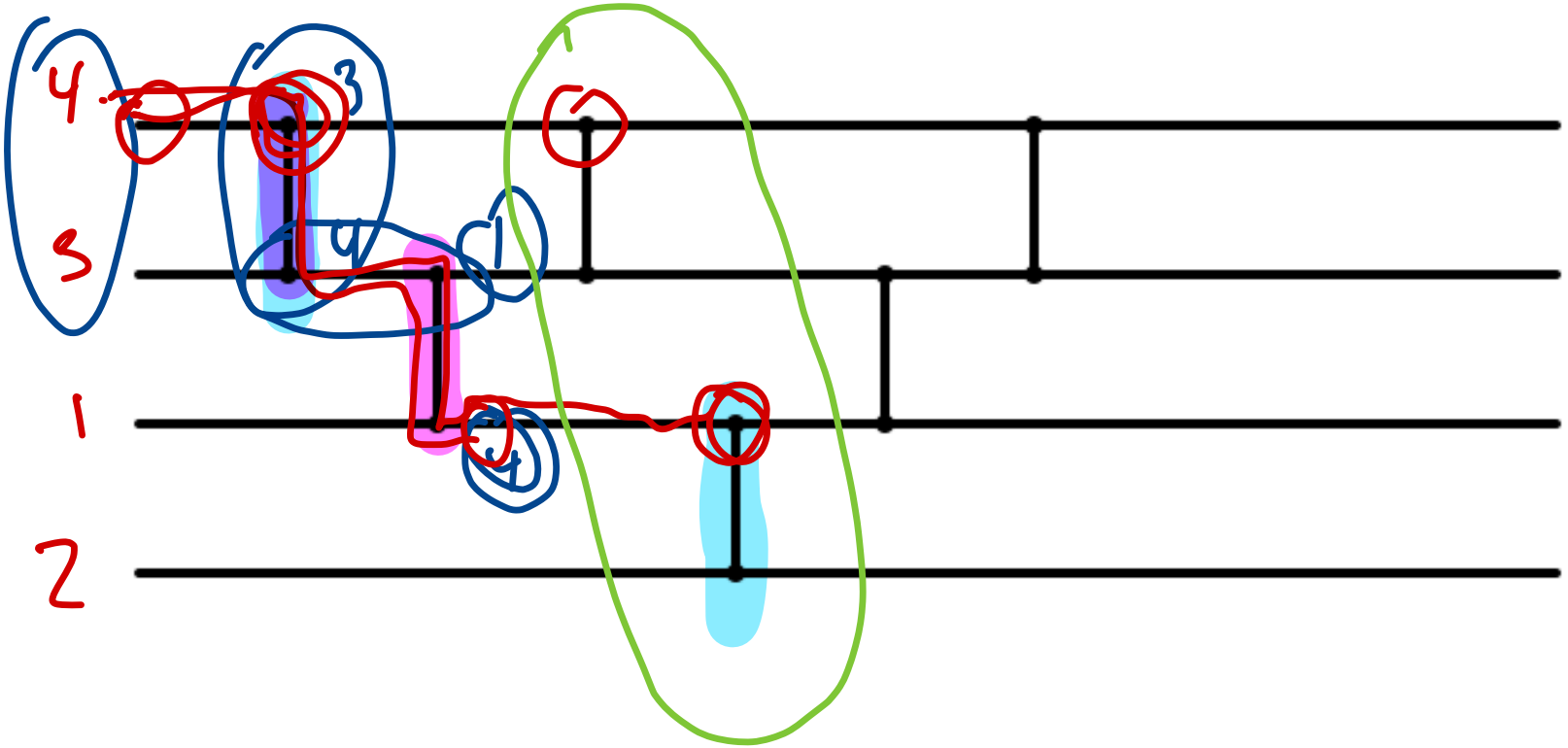
Insertion Sort: `i = 2`



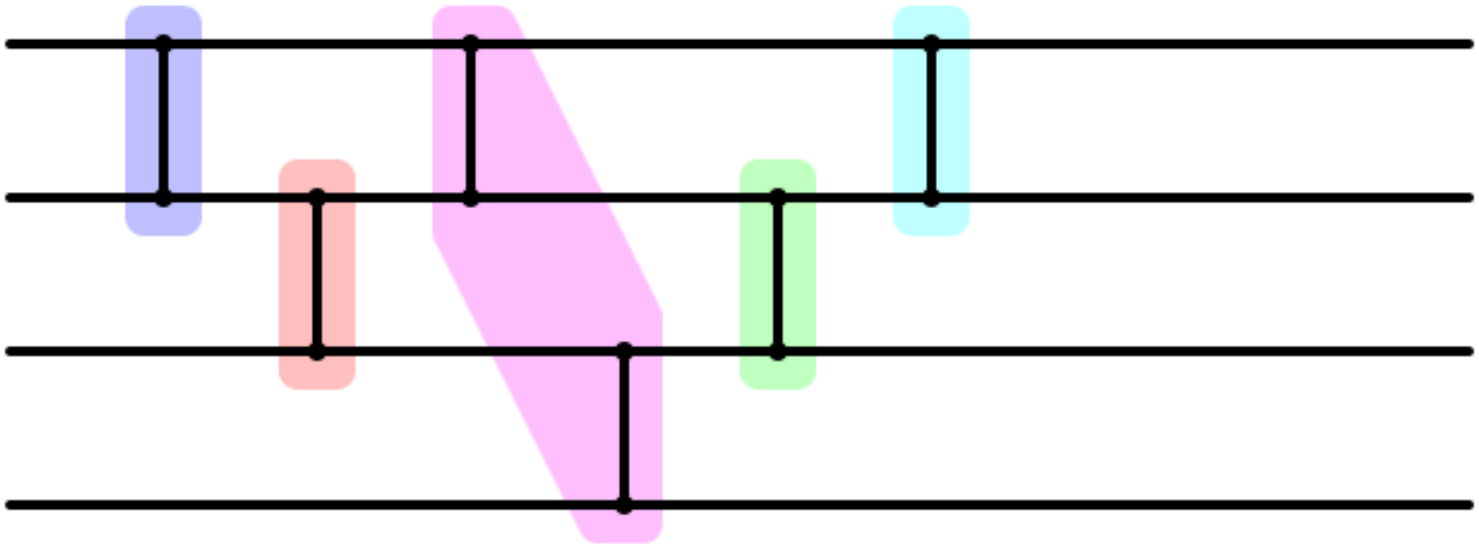
Insertion Sort: `i = 3`



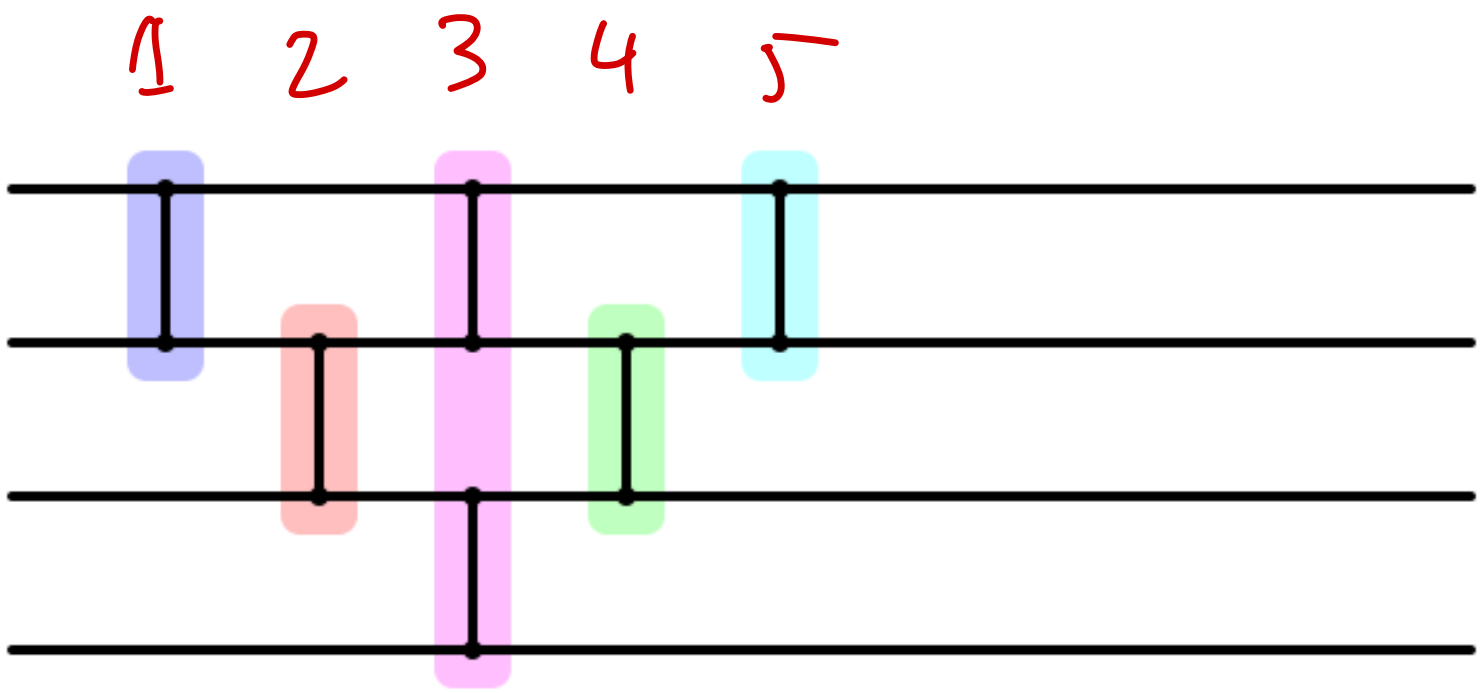
Which Operations can be Parallelized?



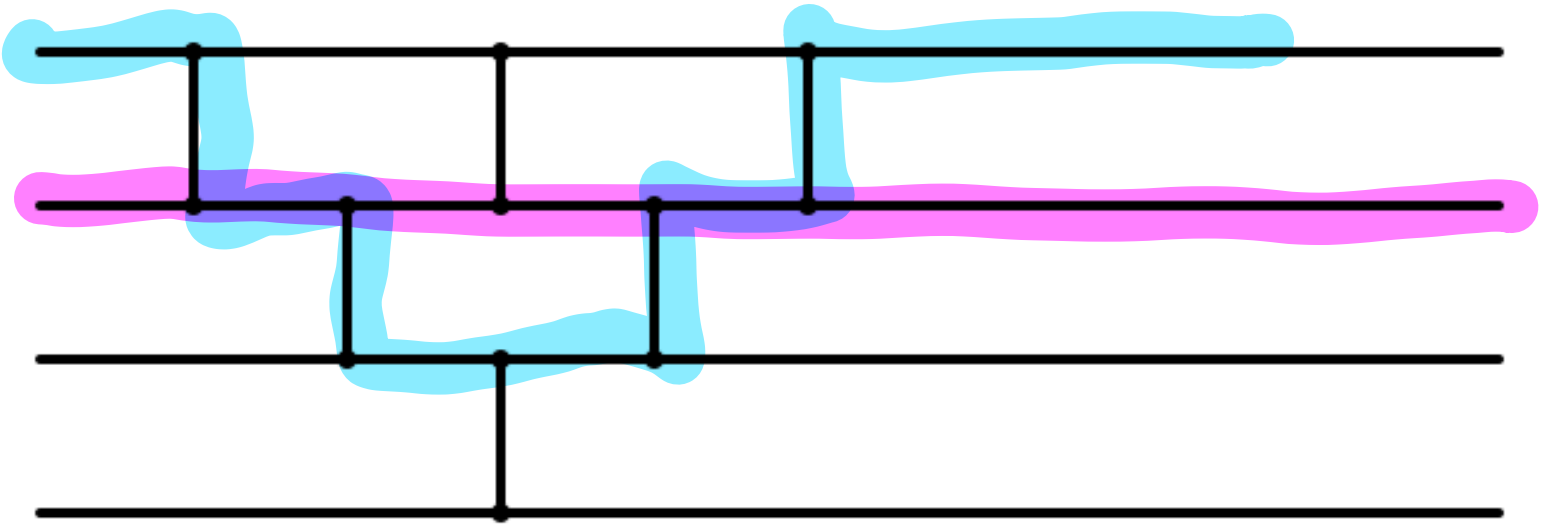
Parallelism



Cleaner Parallel Representation



Depth = length of longest path from input to output

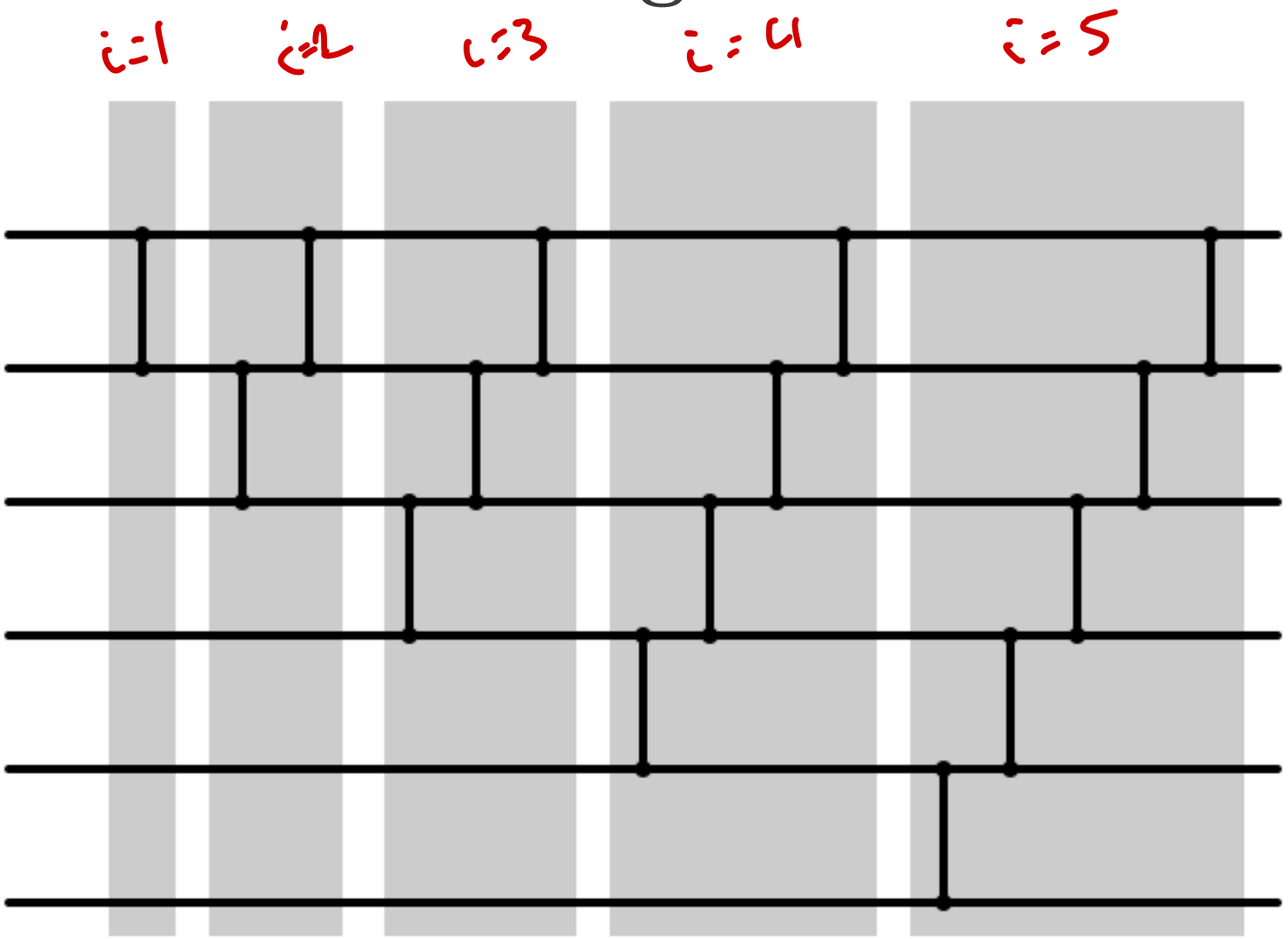


Path of length 5 (longest)

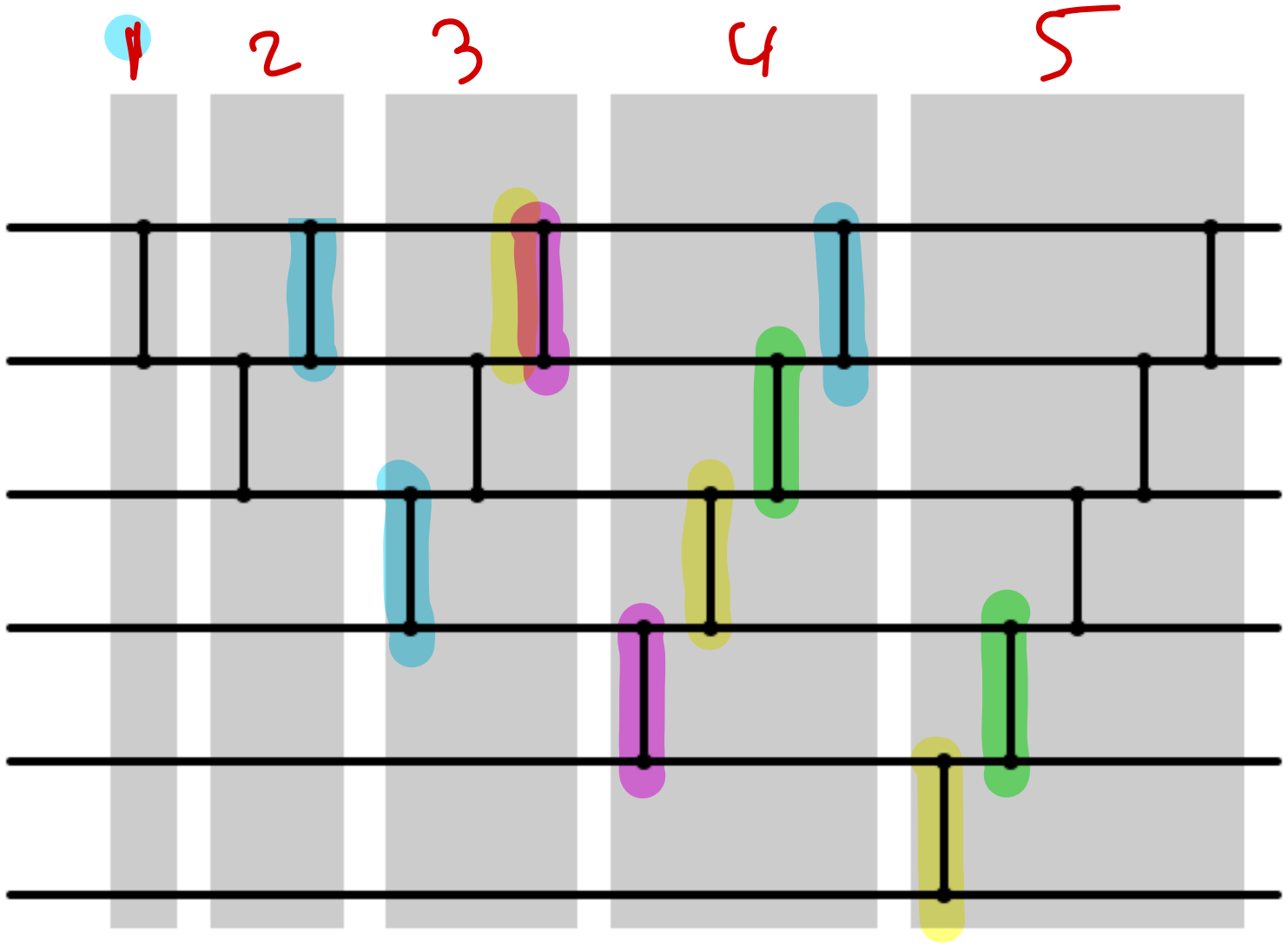
length = # of comparators you touch

Depth = min # of parallel steps in applying swaps

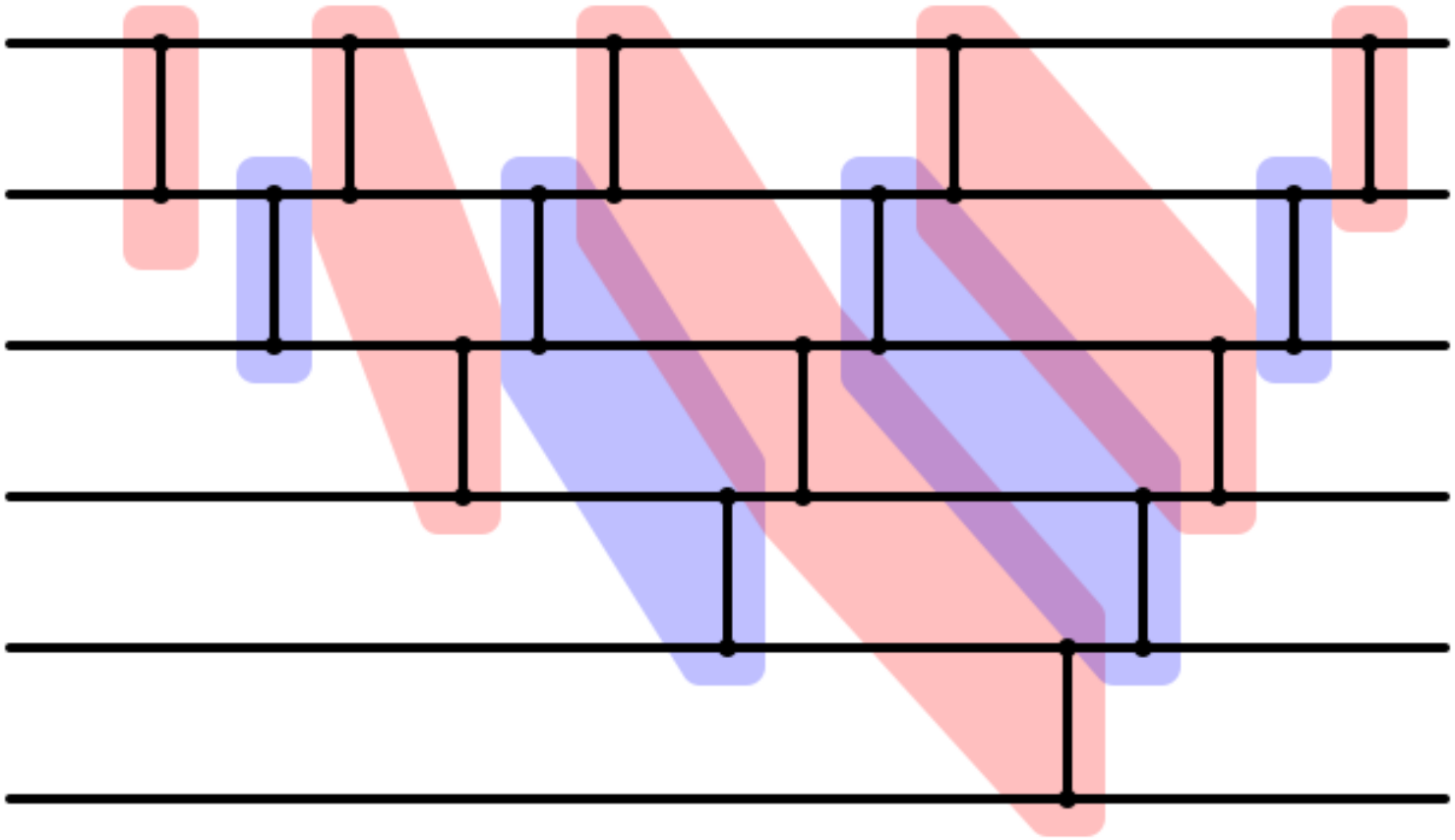
Insertion Sort: Larger Instance



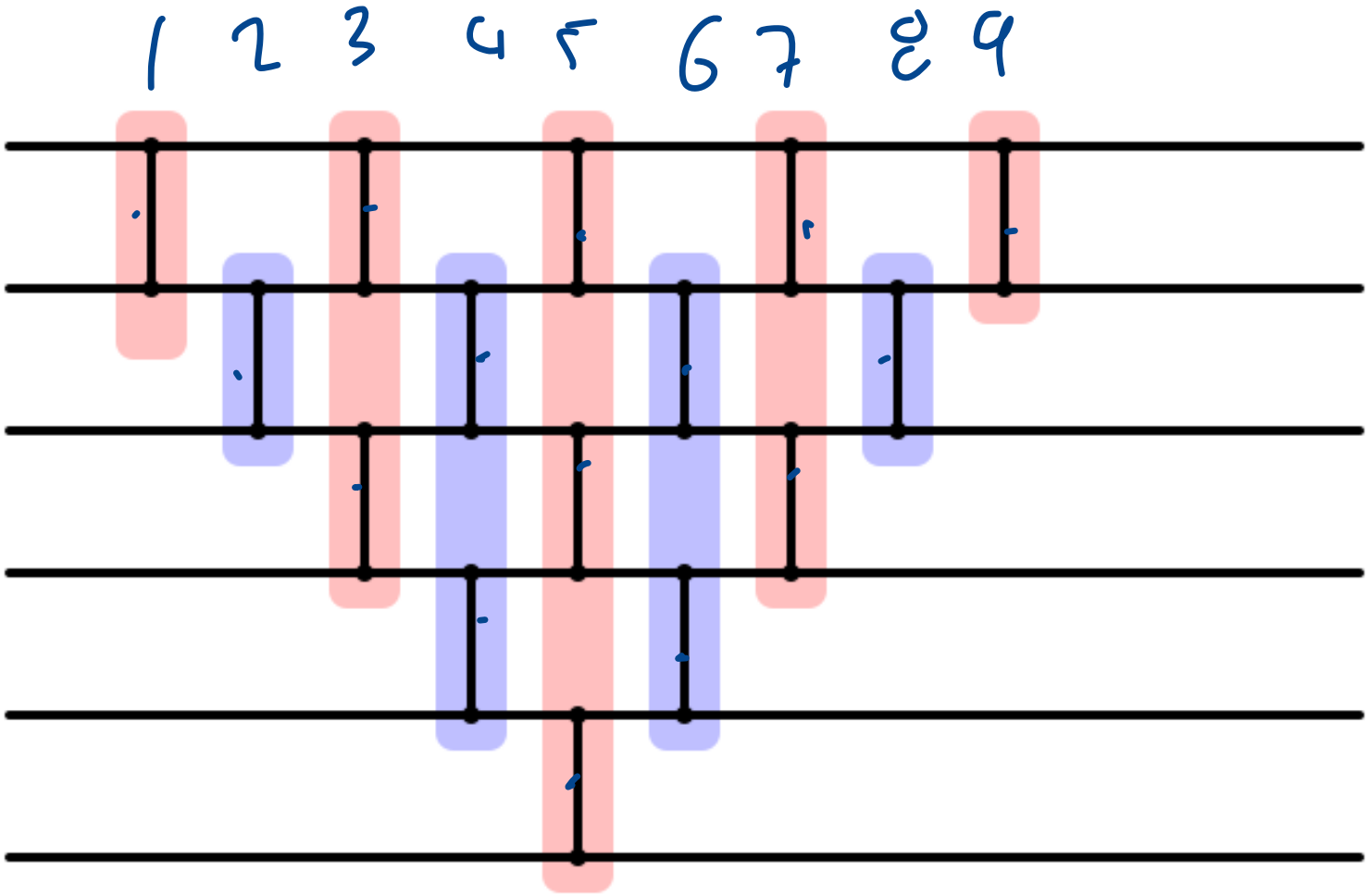
Done In Parallel?



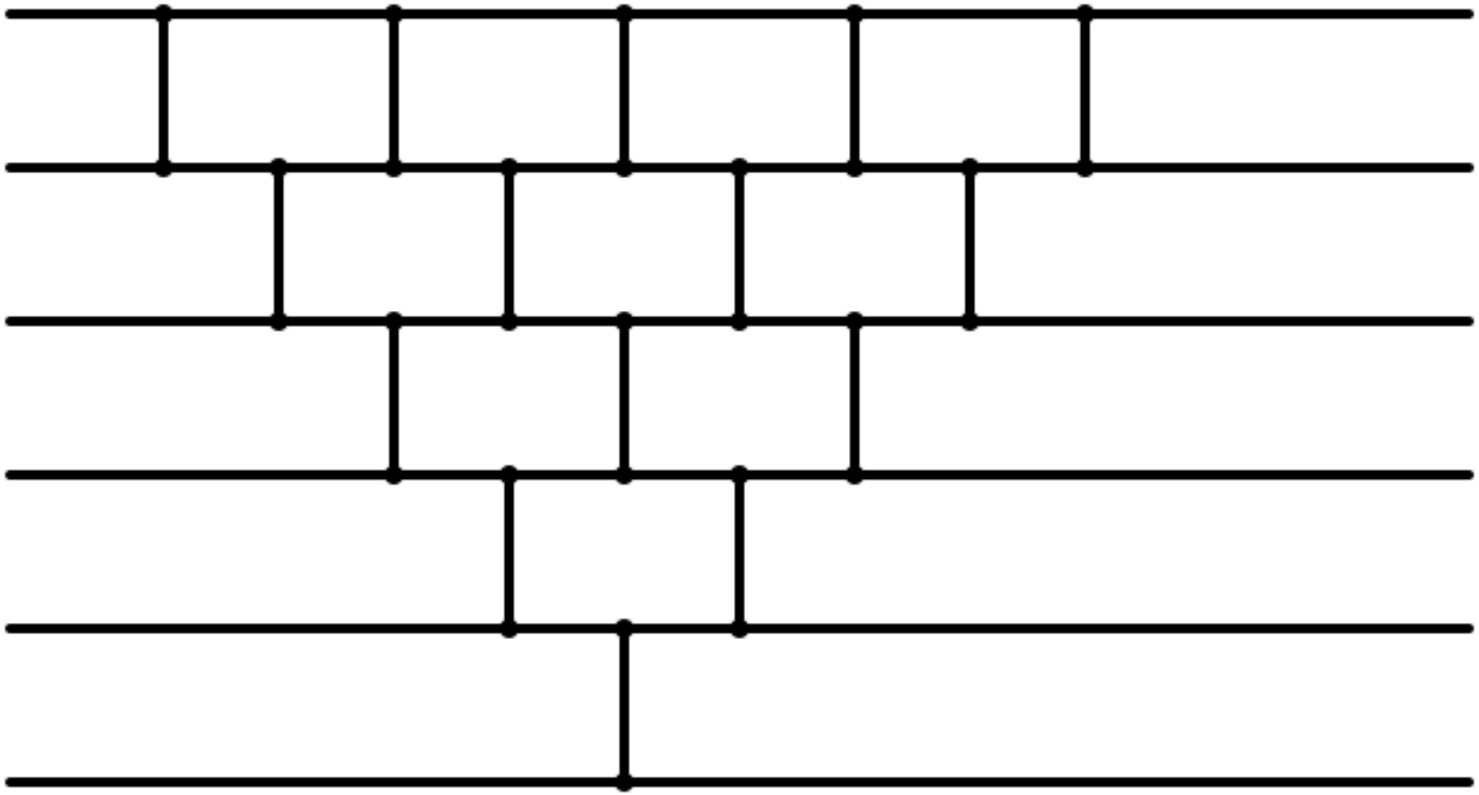
Done In Parallel!



Cleaned Up



Parallel Depth? 9

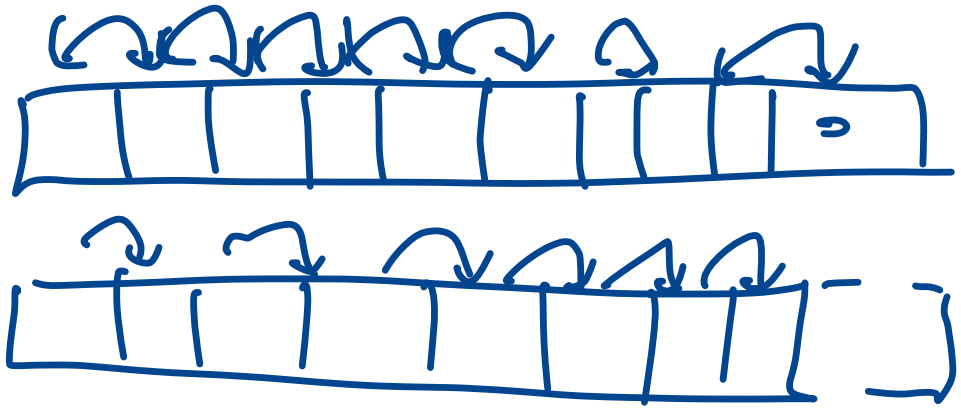


Bubble Sort

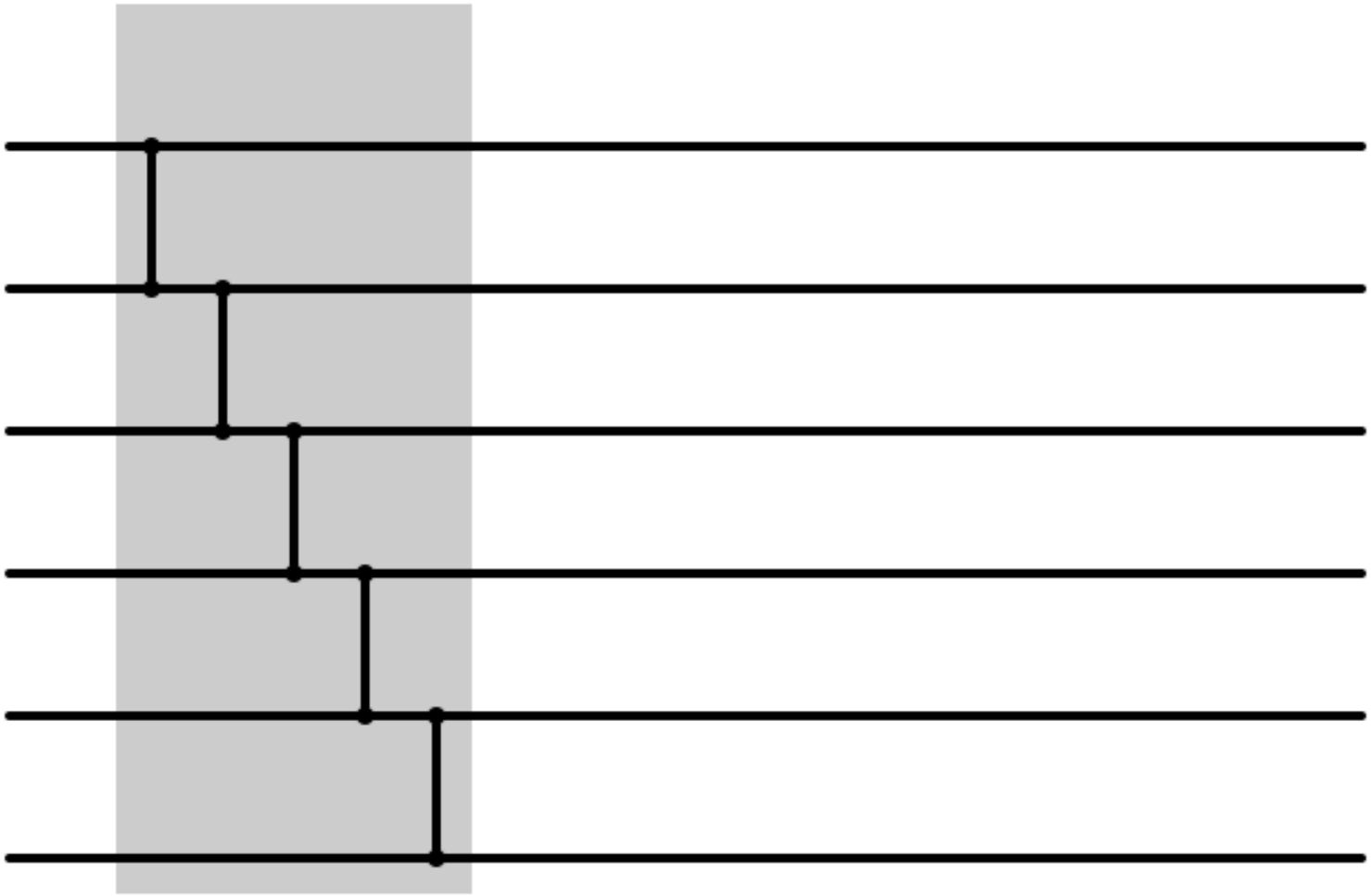
Consider:

```
for (int m = data.length - 1; m > 0; --m) {
    for (int i = 0; i < m; ++i) {
        if (data[i] > data[i+1]) {
            swap(data, i, i+1)
        }
    }
}
```

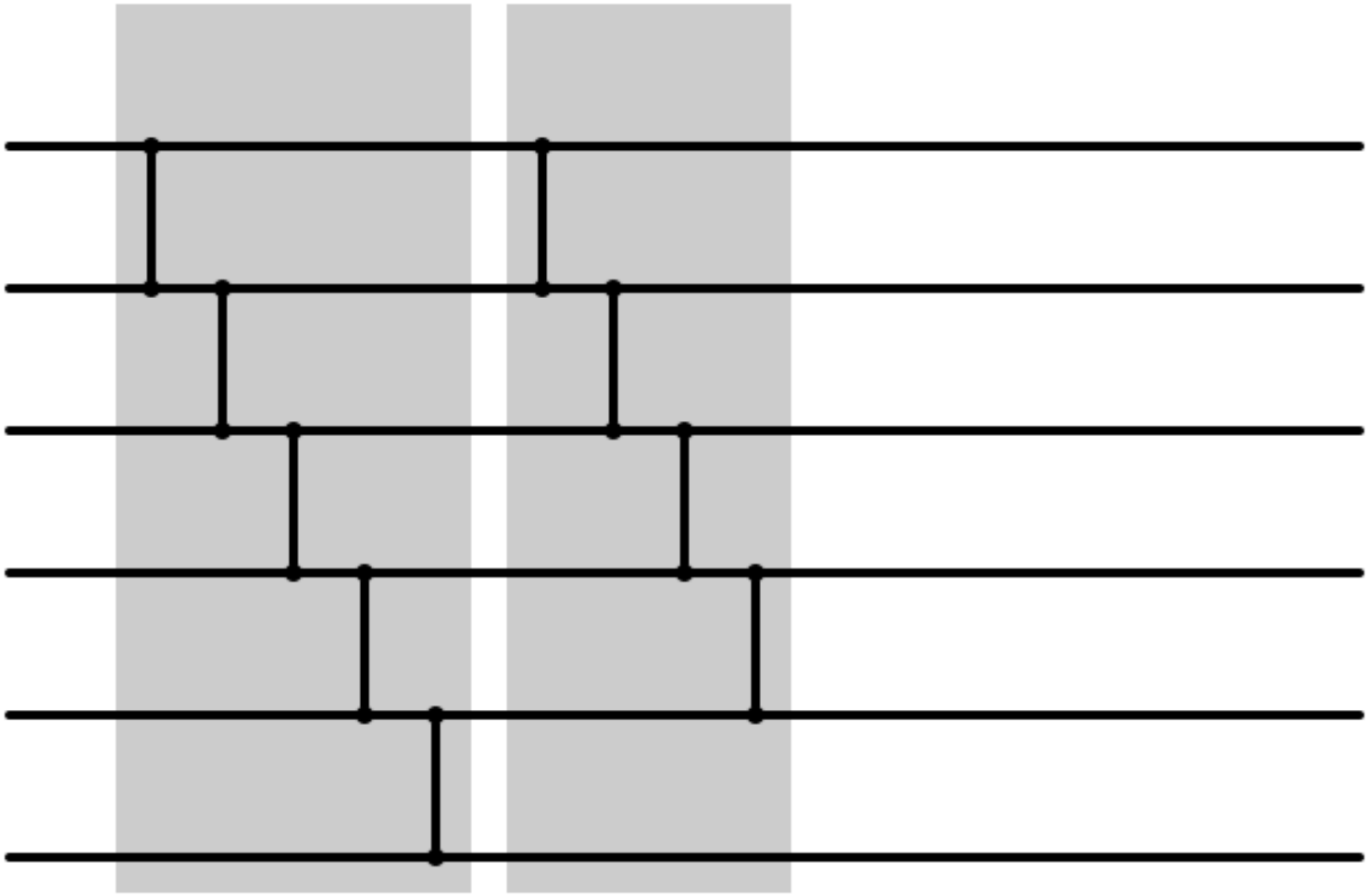
Can we make a sorting network corresponding to bubble sort?



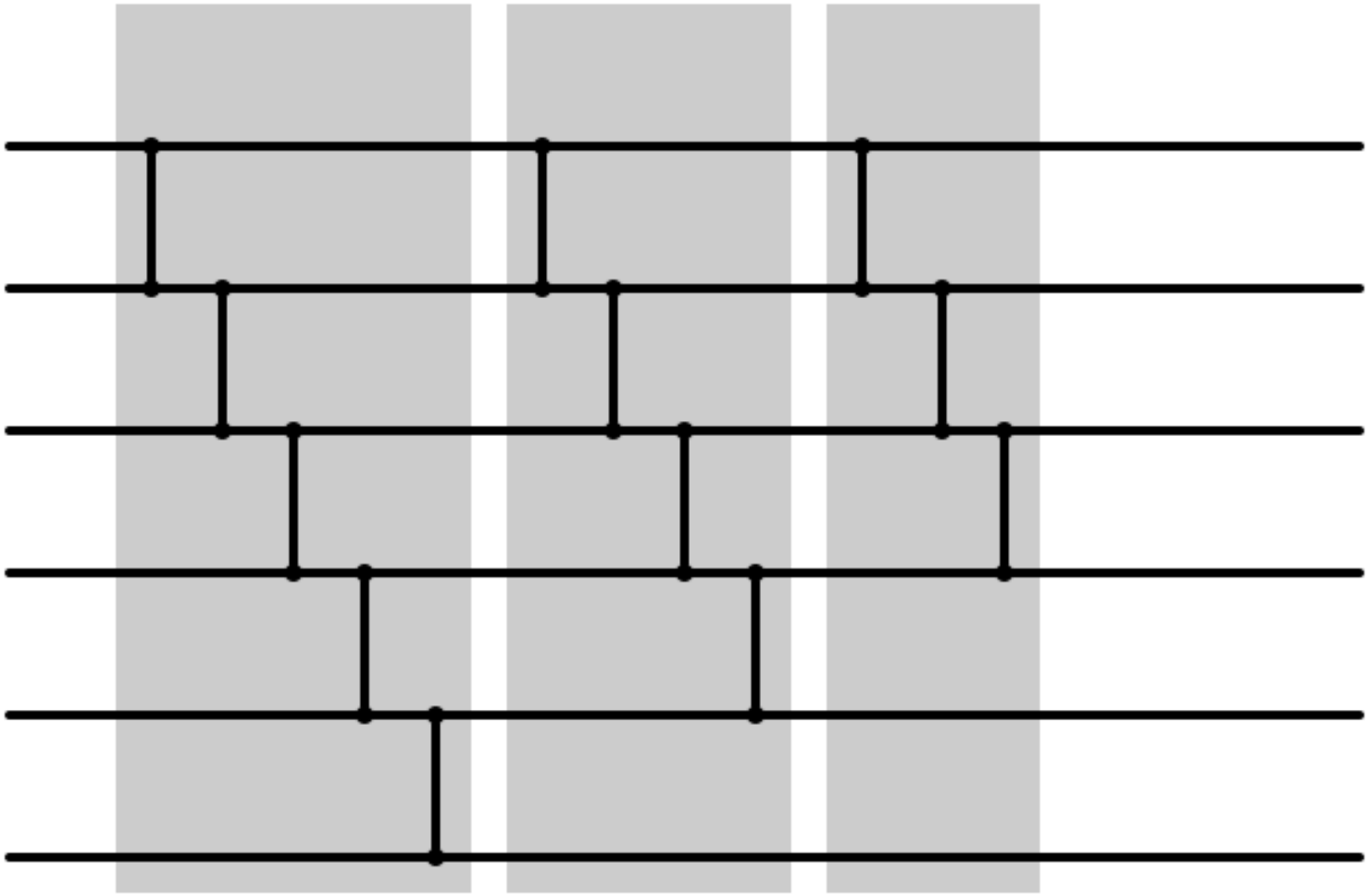
Bubble Sort: `m = 6`



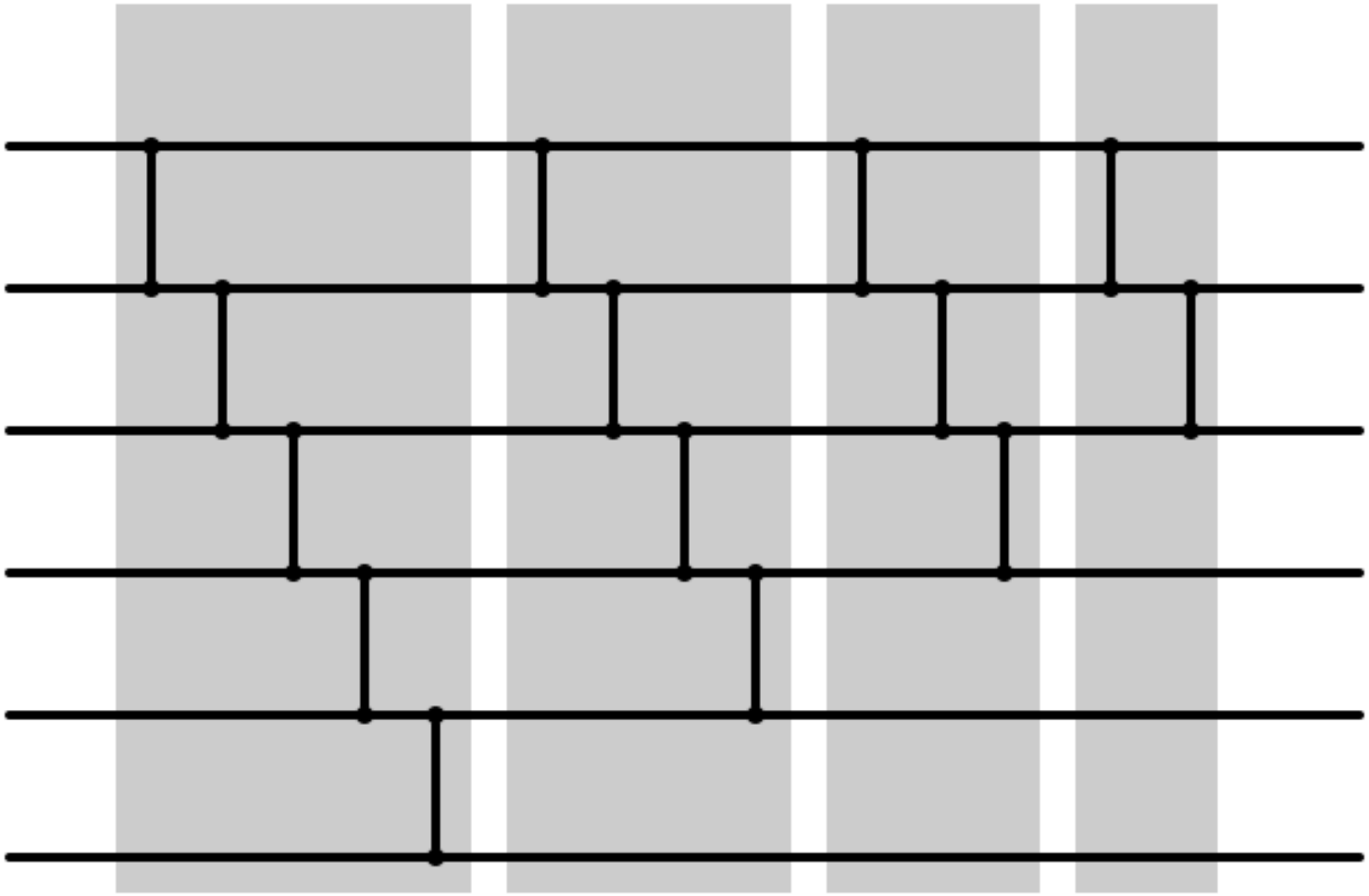
Bubble Sort: `m = 5`



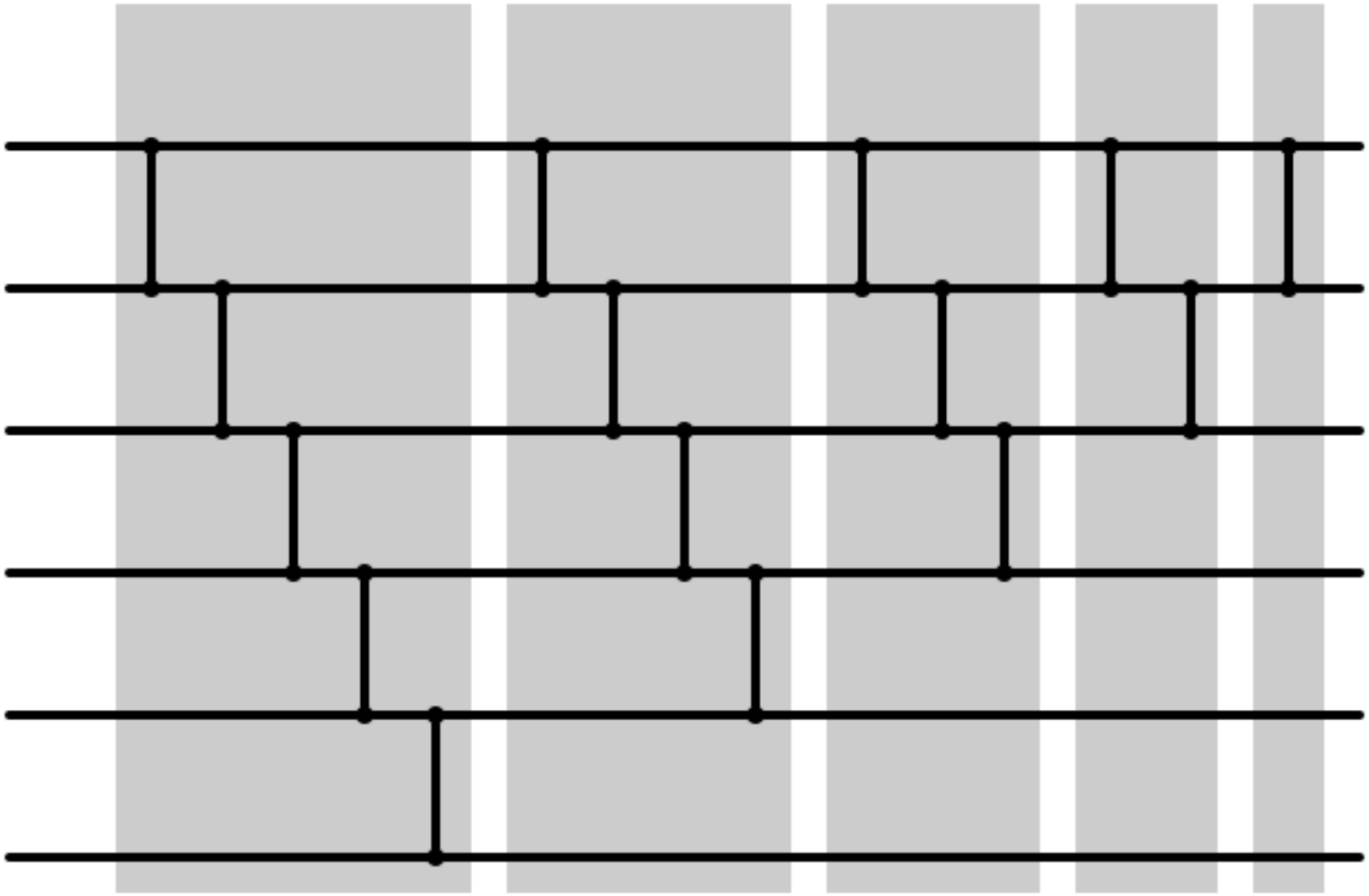
Bubble Sort: `m = 4`



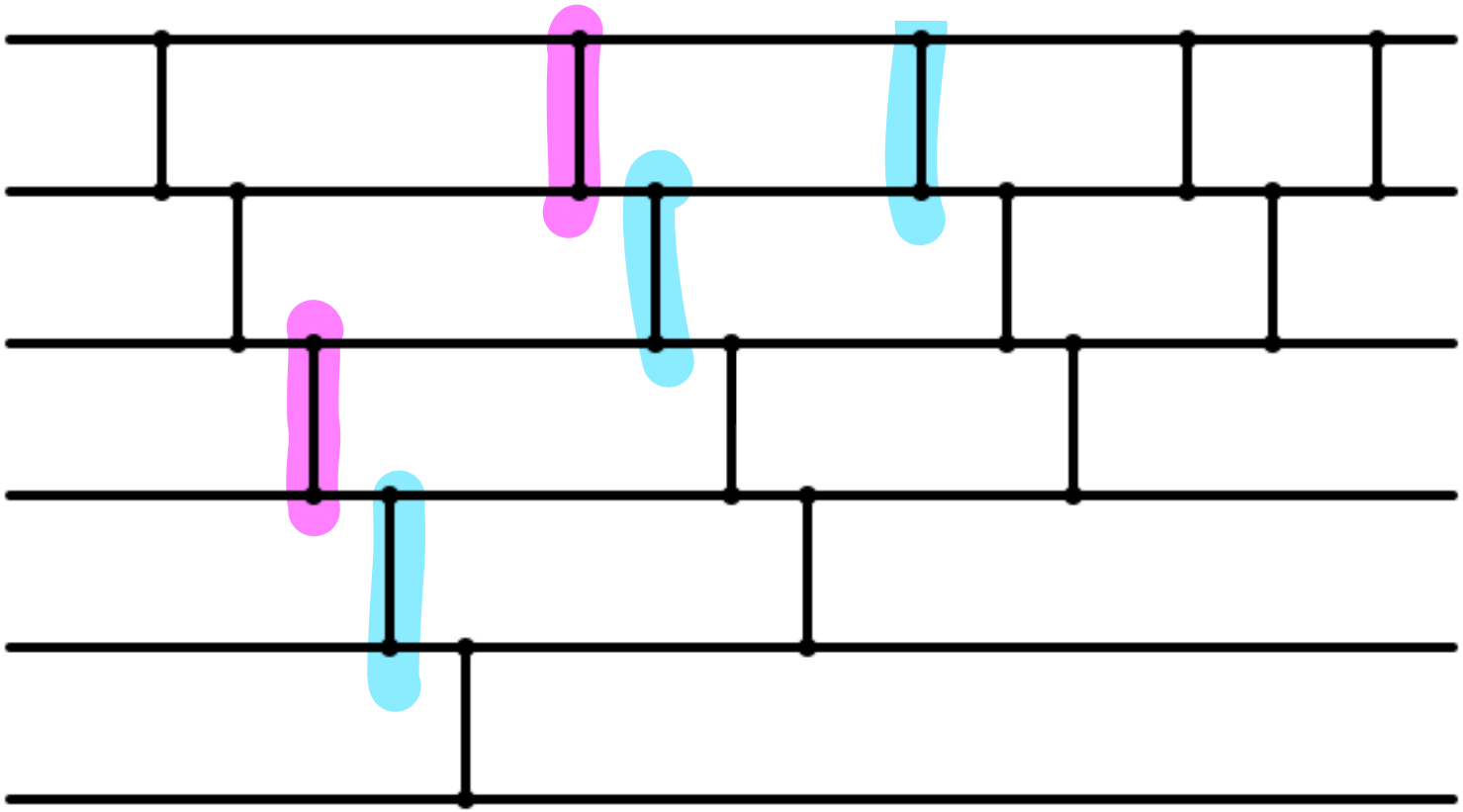
Bubble Sort: `m = 3`



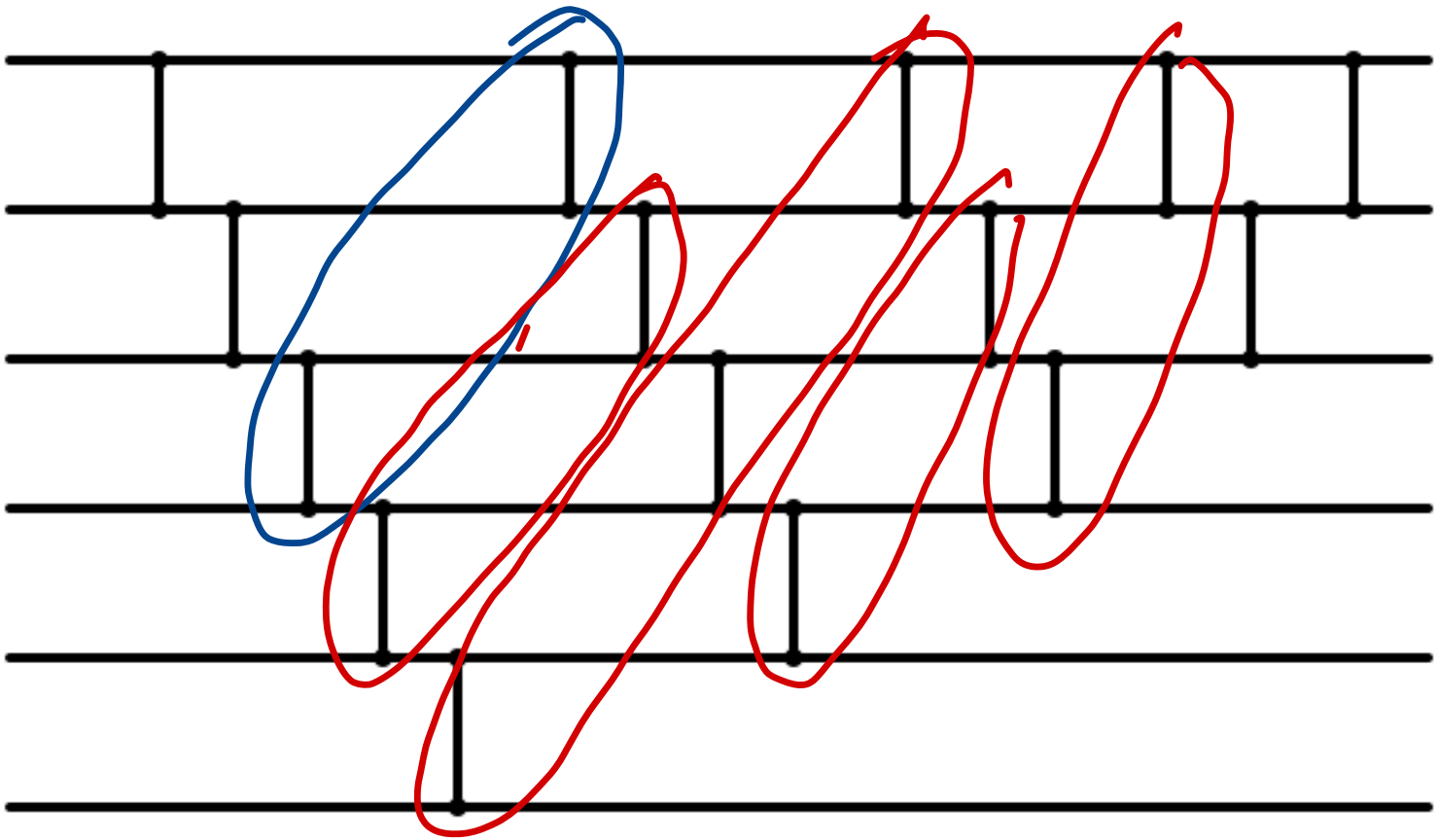
Bubble Sort: `m = 2`



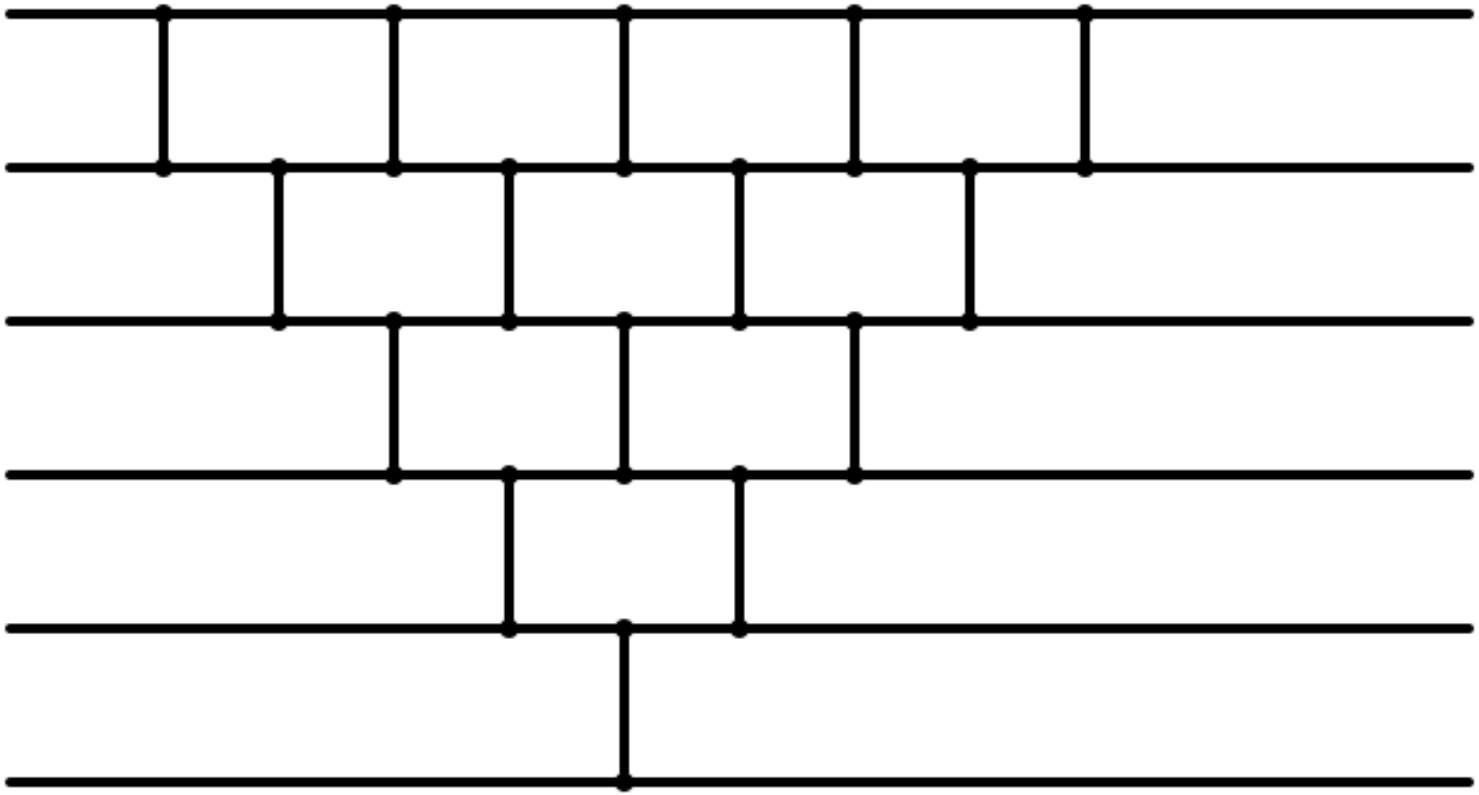
Bubble Sort Network



Bubble Sort Parallelized?



Does it Look Familiar?



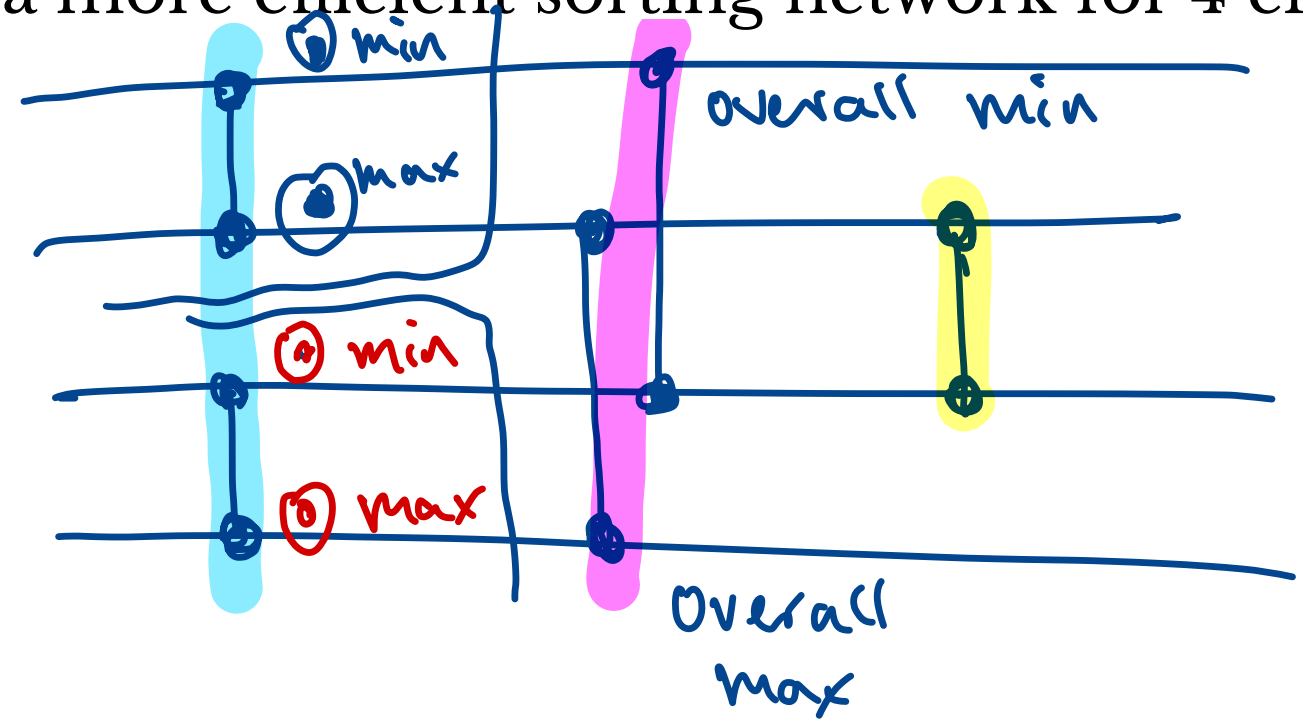
Same as Insertion Sort

Huh

- Insertion sort and bubble sort perform precisely same operations
  - only differ in the order in which comparisons are made
- When fully parallelized, both are same sorting network
- Parallel versions are reasonably efficient
  - depth $2(n-1)-1 = 2n-3$

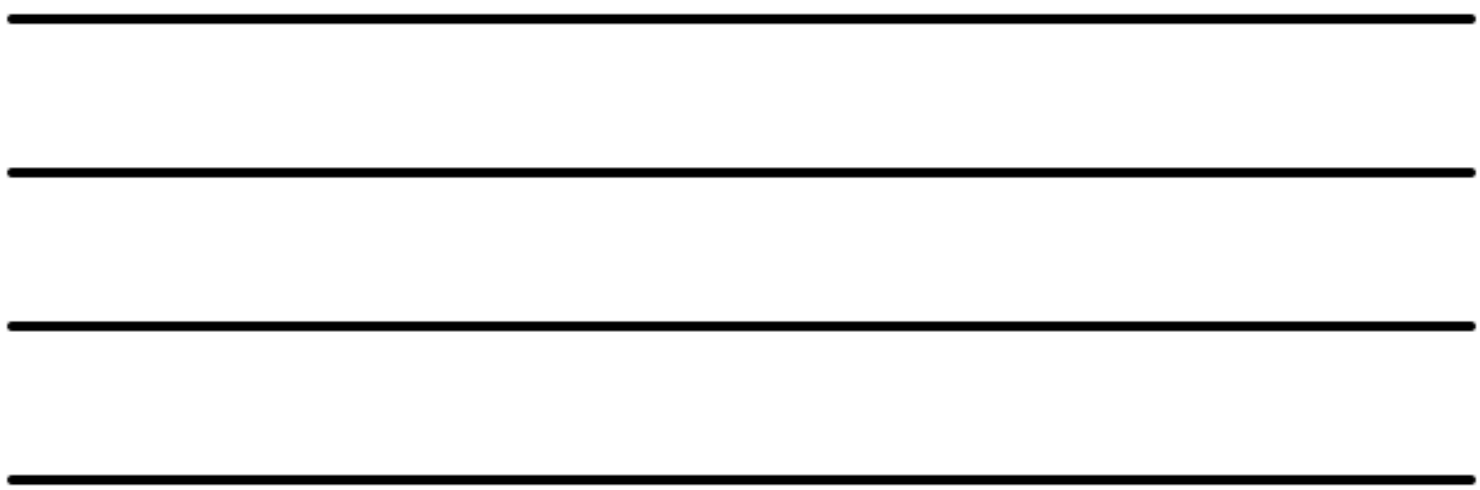
Activity

Make a more efficient sorting network for 4 elements



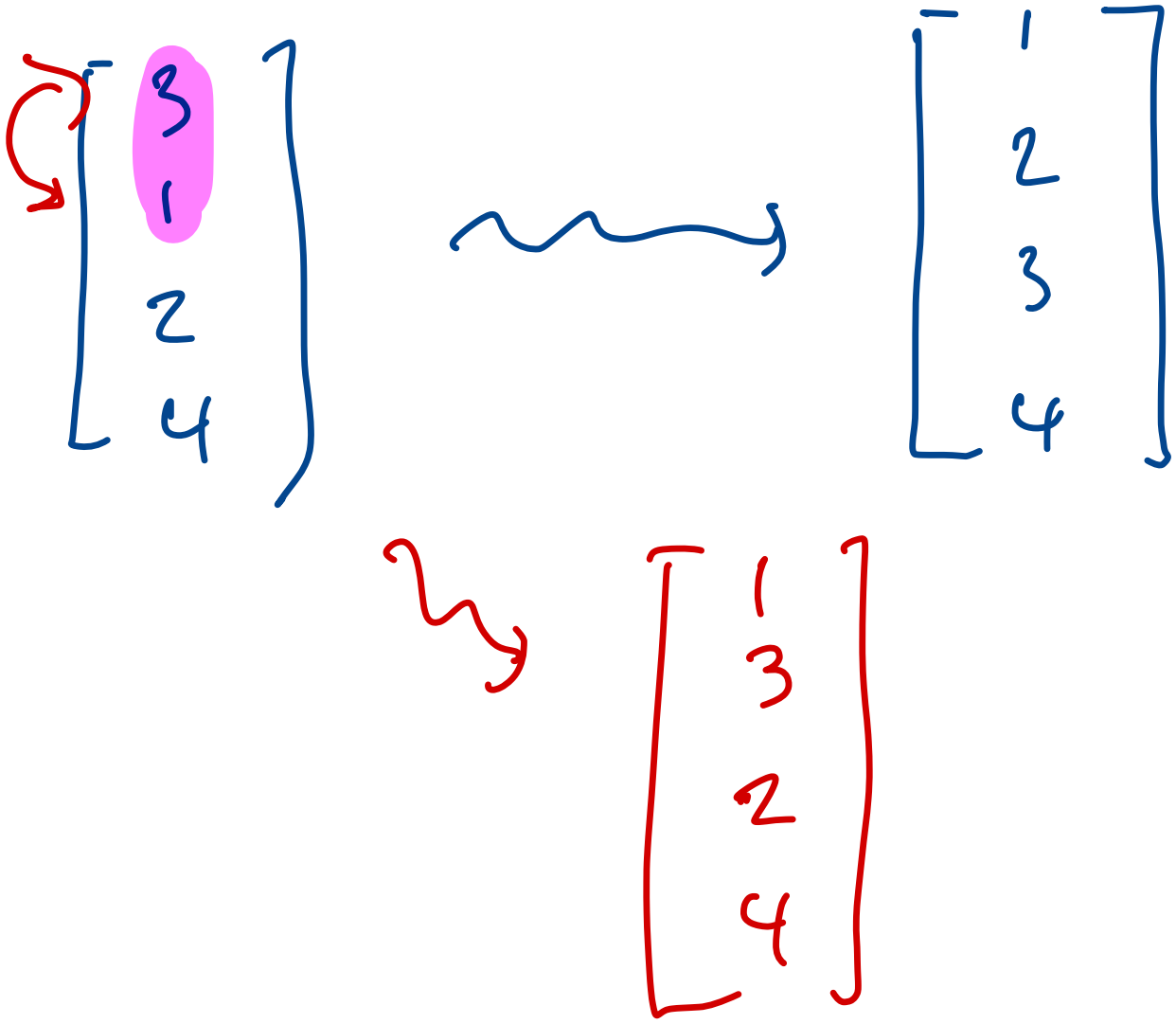
Depth : 3

Sorting Network for 4 Elements?



Sorting `Vectors`

How could we use this sorting network to sort `Vectors` with 4 lanes?



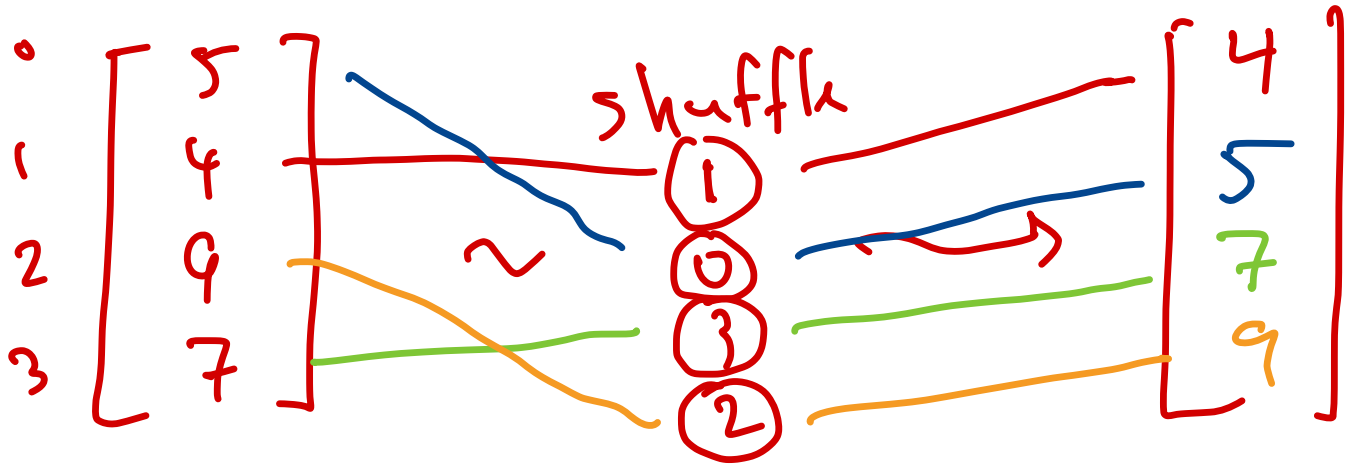
Sorting `Vectors`

How could we use this sorting network to sort `Vectors` with 4 lanes?

A new tool: `VectorShuffle`

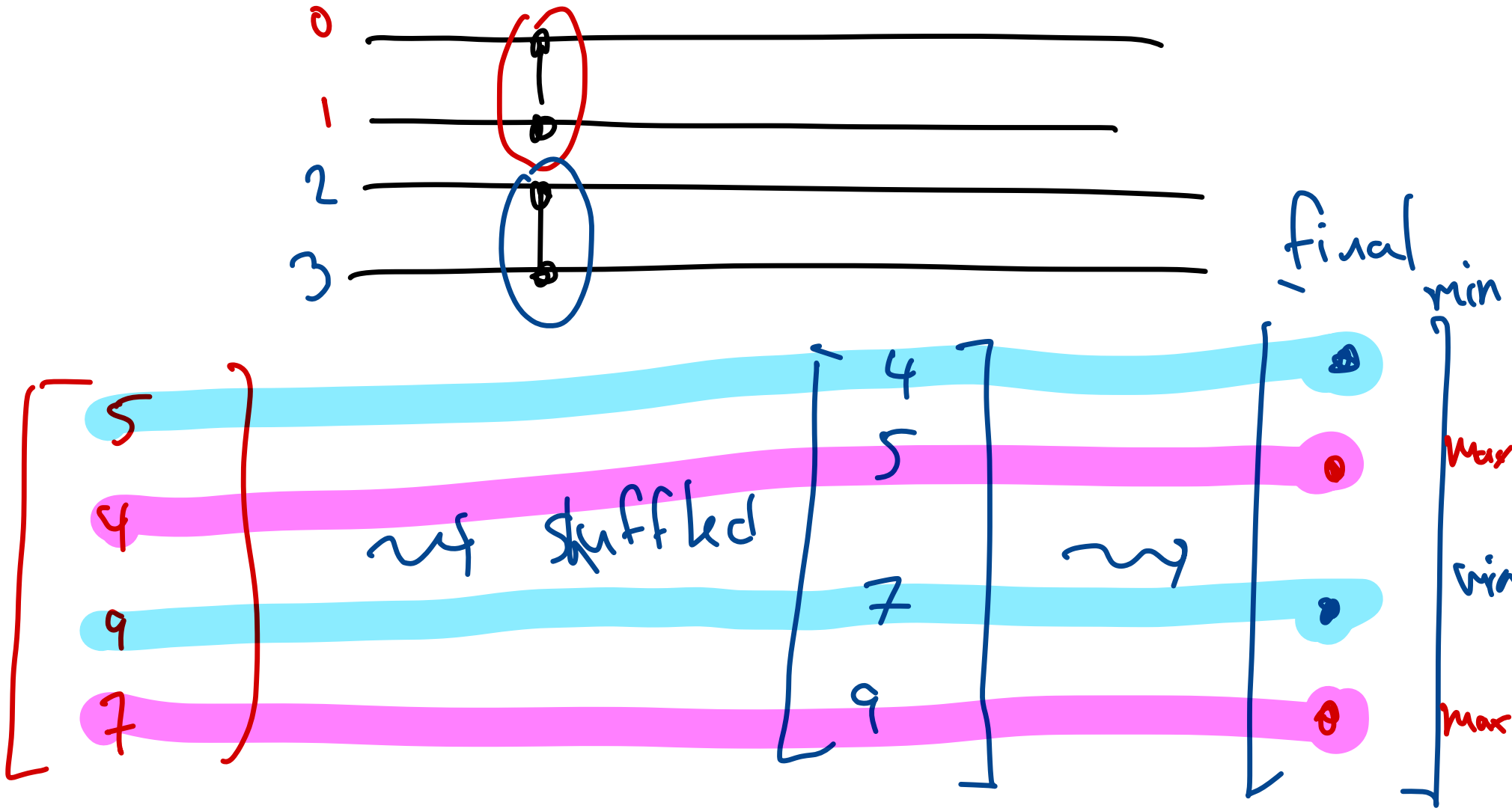
- `VectorShuffle<Float> vs` stores an array of *indices*
  - e.g., `vs` stores `[1, 0, 3, 2]`
- If `vec` is `FloatVector`, `vec.rearrange(vs)`
  - e.g., `vec` stores `[5, 4, 9, 7]`

Question. What is the result of `vec.rearrange(vs)`?



Implementation

How to implement the following parallel comparitor operations?



Example, In Pictures

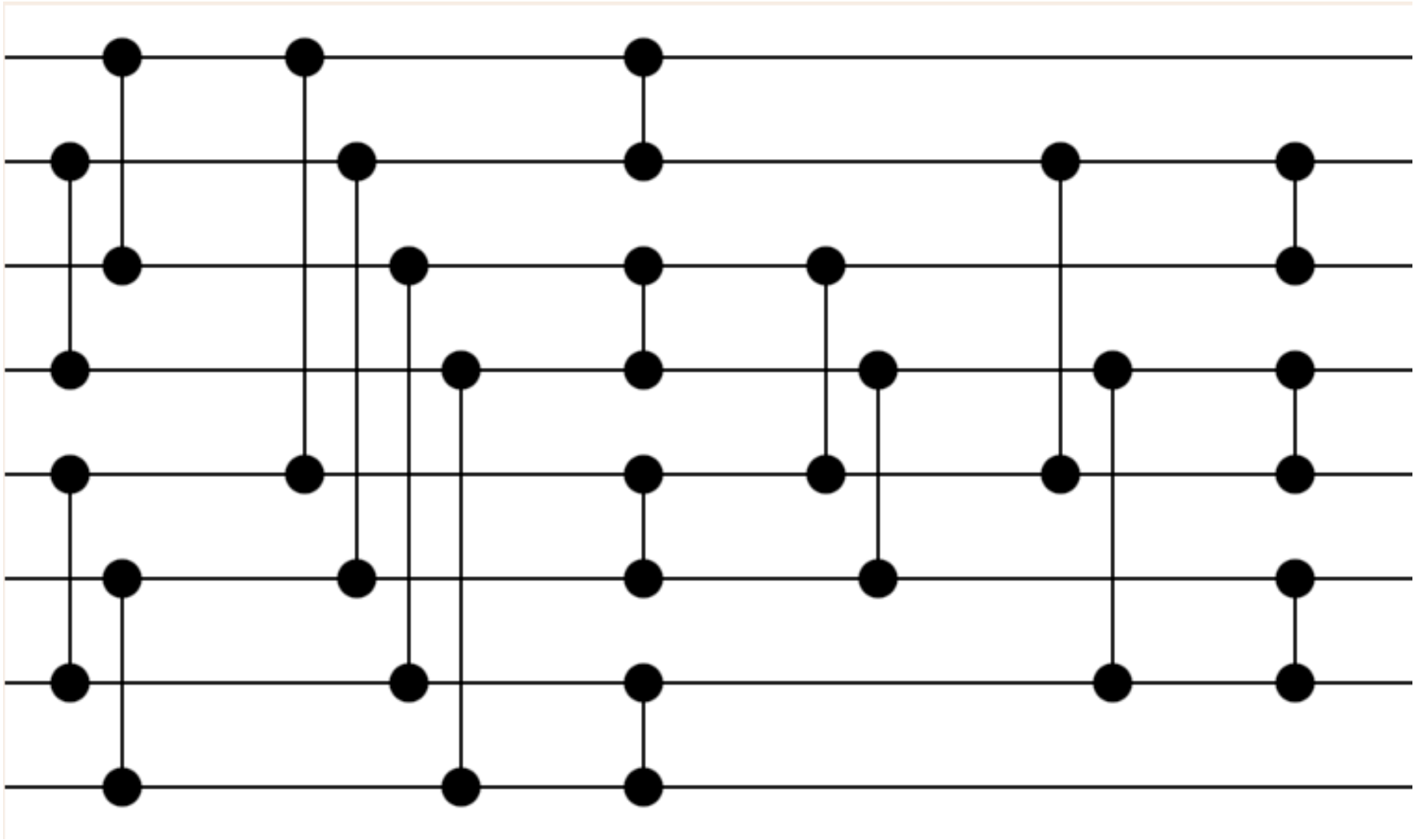
- `vec = [7, 4, 5, 6]`, `vs = [1, 0, 3, 2]`

Example, In Code

Original vector `vec`, shuffle `vs`, mask `mask` is `true` for all lanes corresponding to `min` comparator

```
var swapped = vec.rearrange(vs);
vec = vec.blend(vec.min(swapped, mask)
          .blend(vec.max(swapped, mask.not()));
```

Optimal Network of Size 8



Testing It!

- `sorting-networks.zip` implements an optimal sorting network of size 8
- compares performance of sorting 1M blocks of size 8
  - vector sorting network vs insertion sort

Next Week

More shared data structures: linked lists!