Lecture 10: Convex Hulls; Animation

COSC 225: Algorithms and Visualization

Spring, 2023

Annoucements

Assignment 06: Submit Pair Preferecnes Today!

Outline

1. Convex Hulls, Finished
2. 3 Ways to Animate!
   - CSS `transition` property
   - `setInterval`
   - `window.requestAnimationFrame`

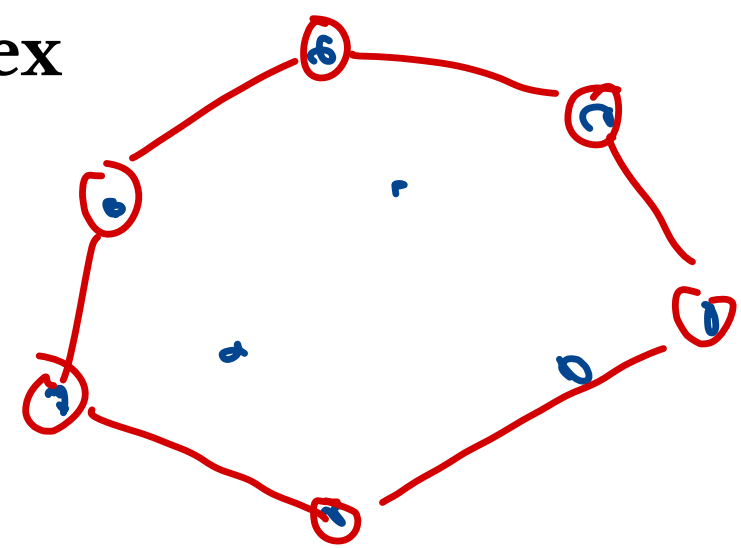
Last Time: Convex Hulls

Input:

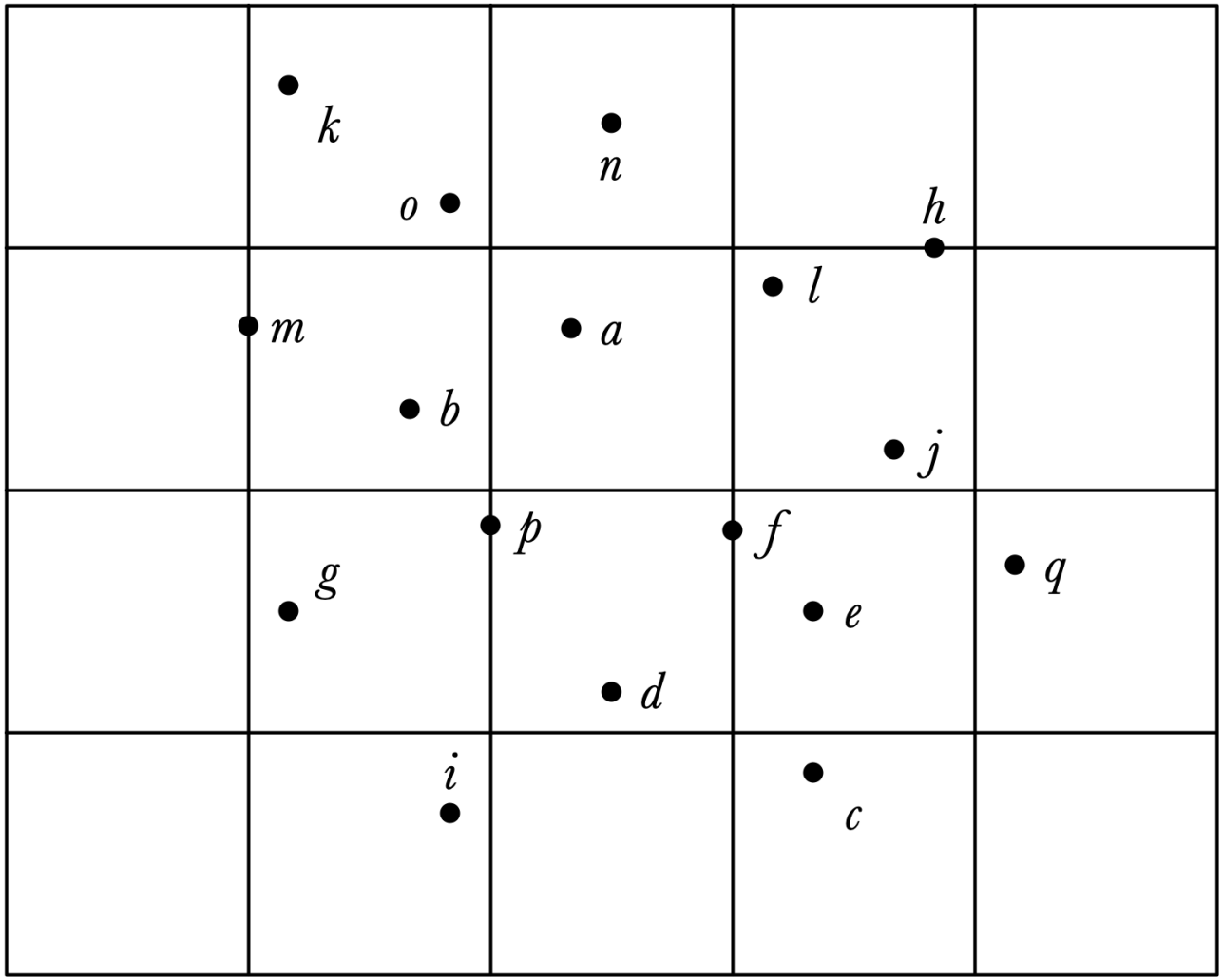
- set of points in plane
  - (x, y)-coordinates of each point

Output:

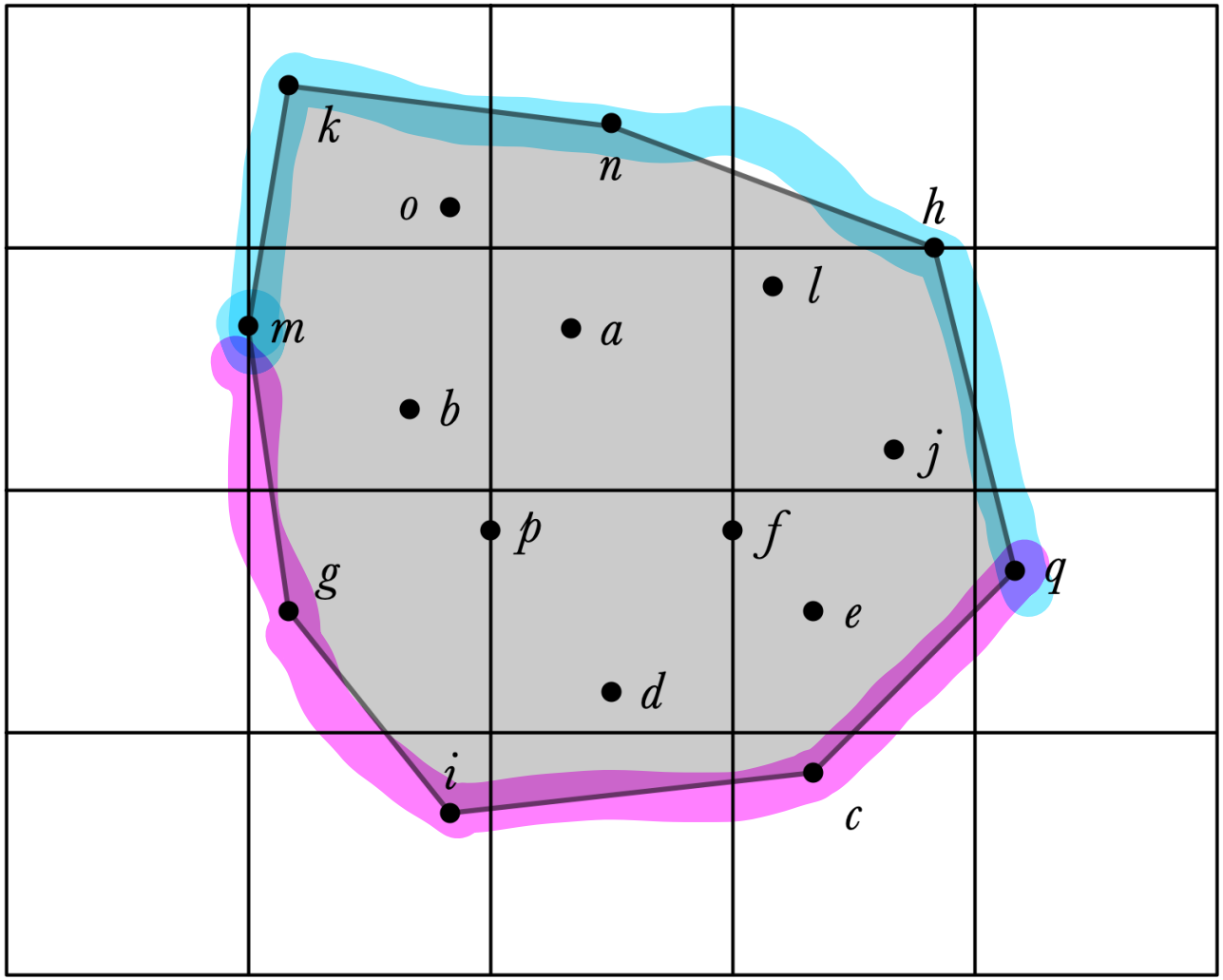
- a sequence of points $(x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k)$ that define the "boundary" of the set of points
  - path around $(x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k)$ surrounds all points in the set in clockwise order
  - the bounded region is **convex**



Input



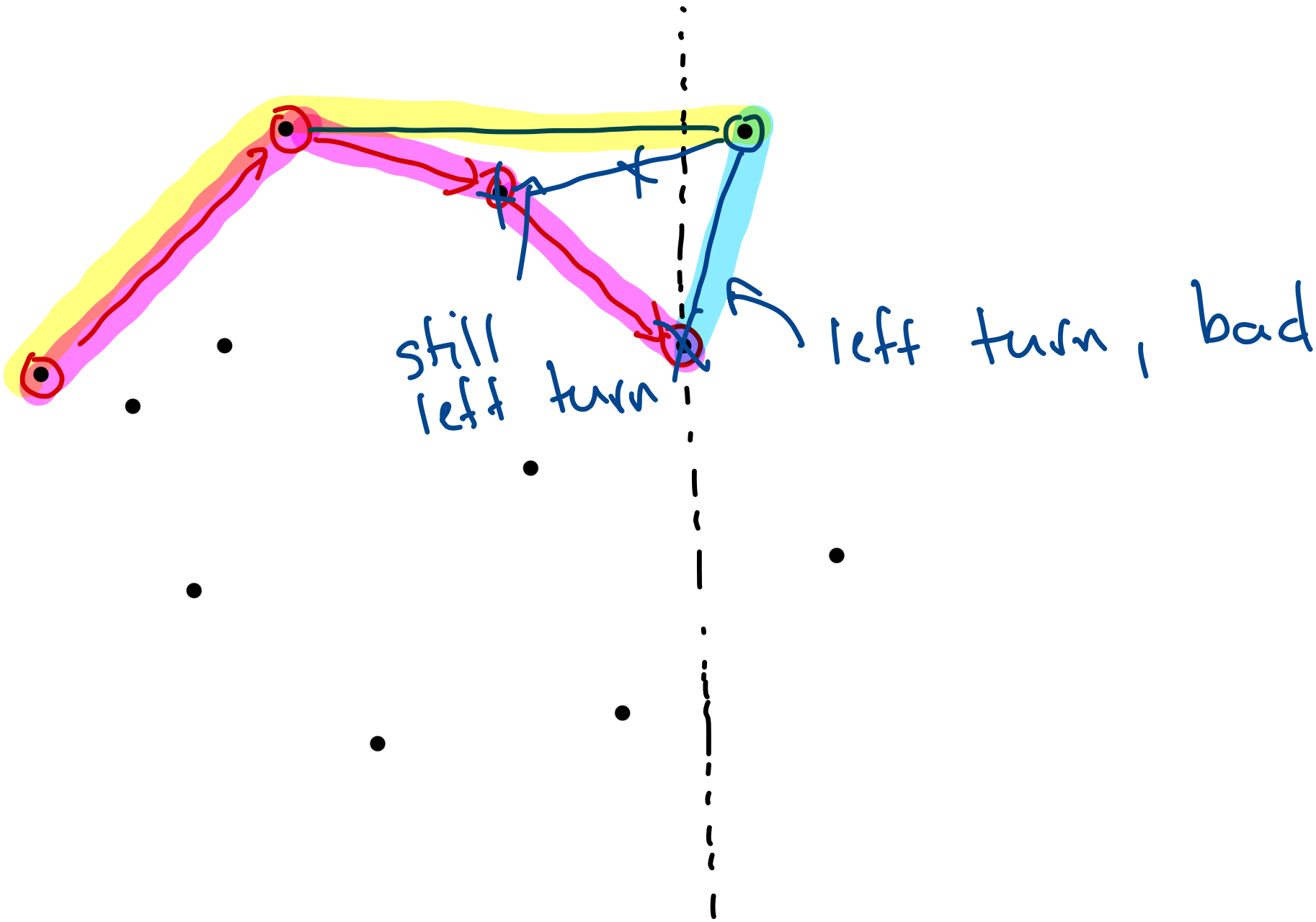
Convex Hull



Grahm's Scan Algorithm

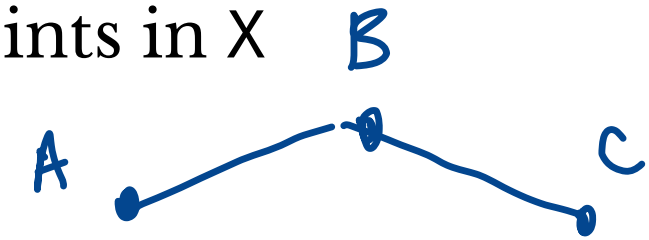
1. Pre-process X by sorting **by x-coordinate:**
2. Consider points one by one to determine if they are on the upper boundary of $CH(X)$
3. repeat process from right to left to get **lower** convex hull

Graham Scan Idea, Illustrated



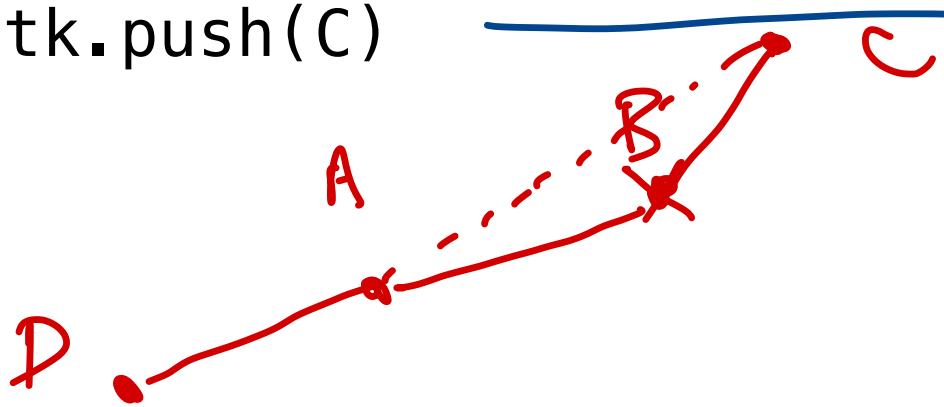
Graham's Scan Pseudocode

- `X` sorted by x-coordinate
- `stk` a stack, initially storing first two points in `X`

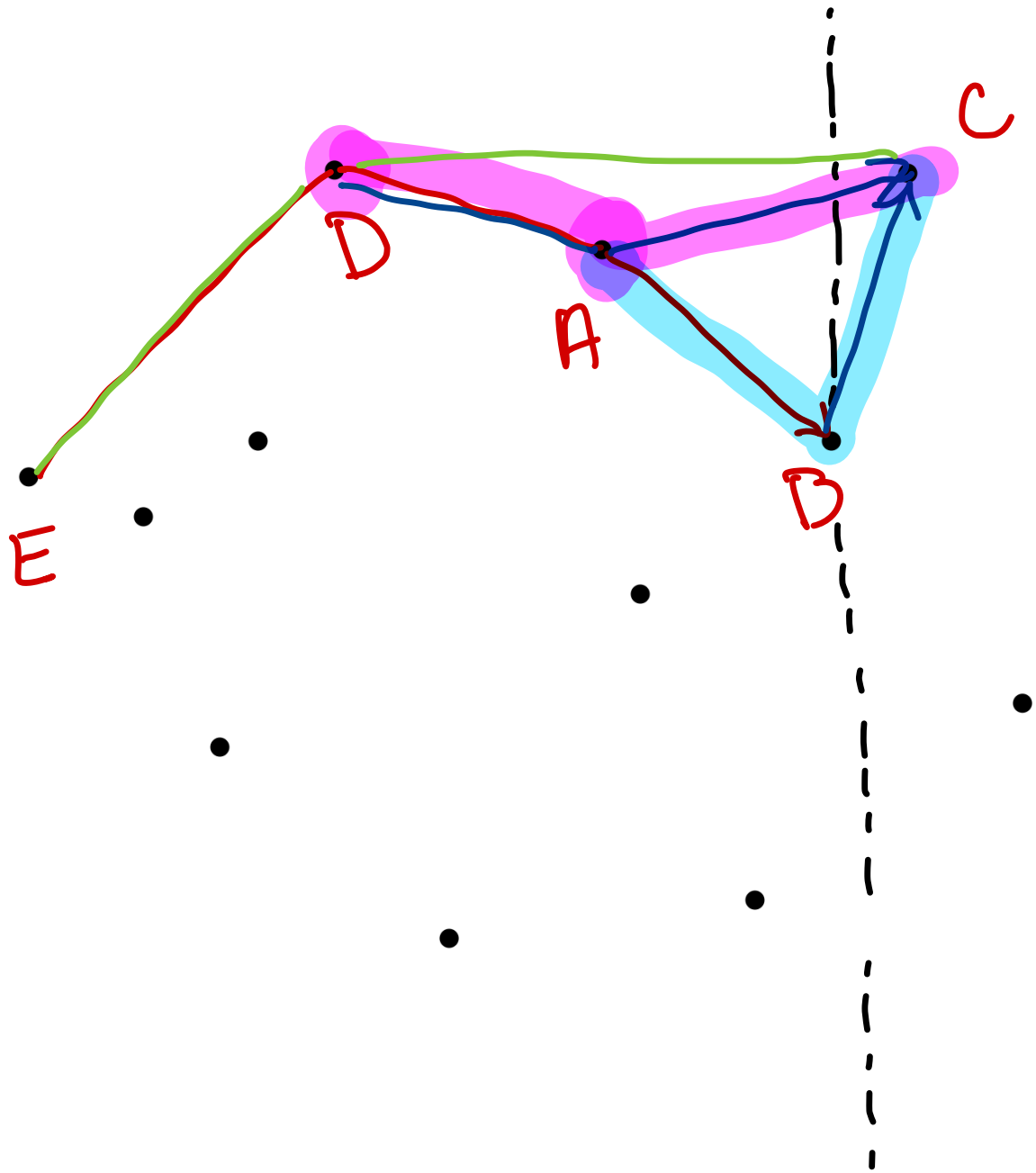


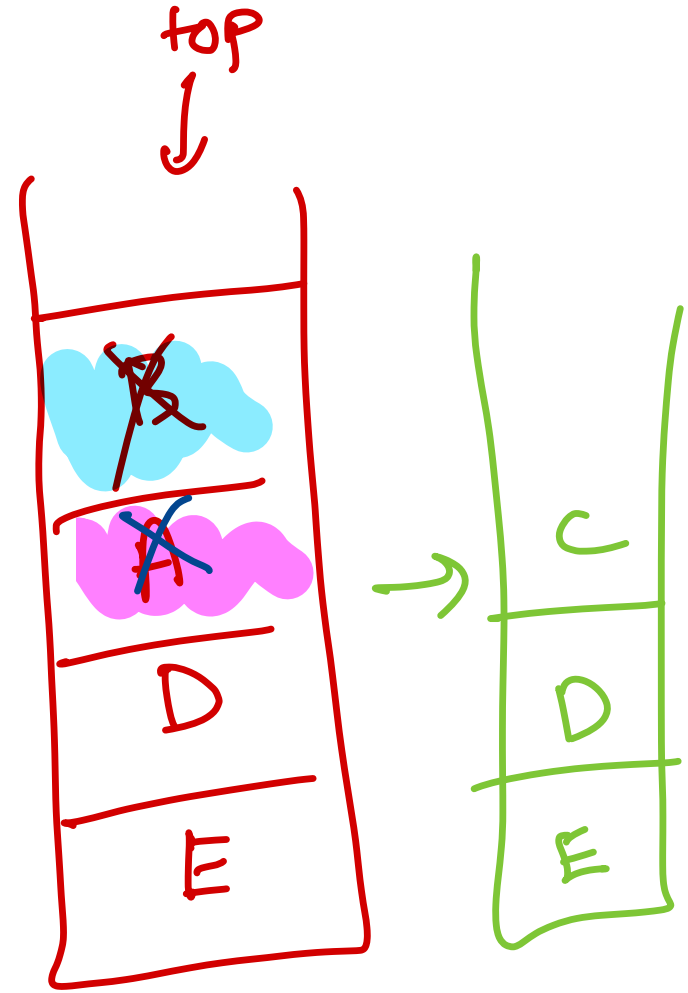
For each remaining `C` in `X`:

- if `stk.size() == 1`, `stk.push(C)`
- otherwise
  - `A` and `B` are top two elements in `stk`
  - while `ABC` is not a right turn and `stk.size() > 1`
    - `stk.pop()`, update `A`, `B`
  - `stk.push(C)`



Graham Scan with a Stack





stk

pop stack

pop stack

push C

Claim

When Graham's Scan completes, `stk` stores the points along the upper boundary of the convex hull of X.

Why?

Claim

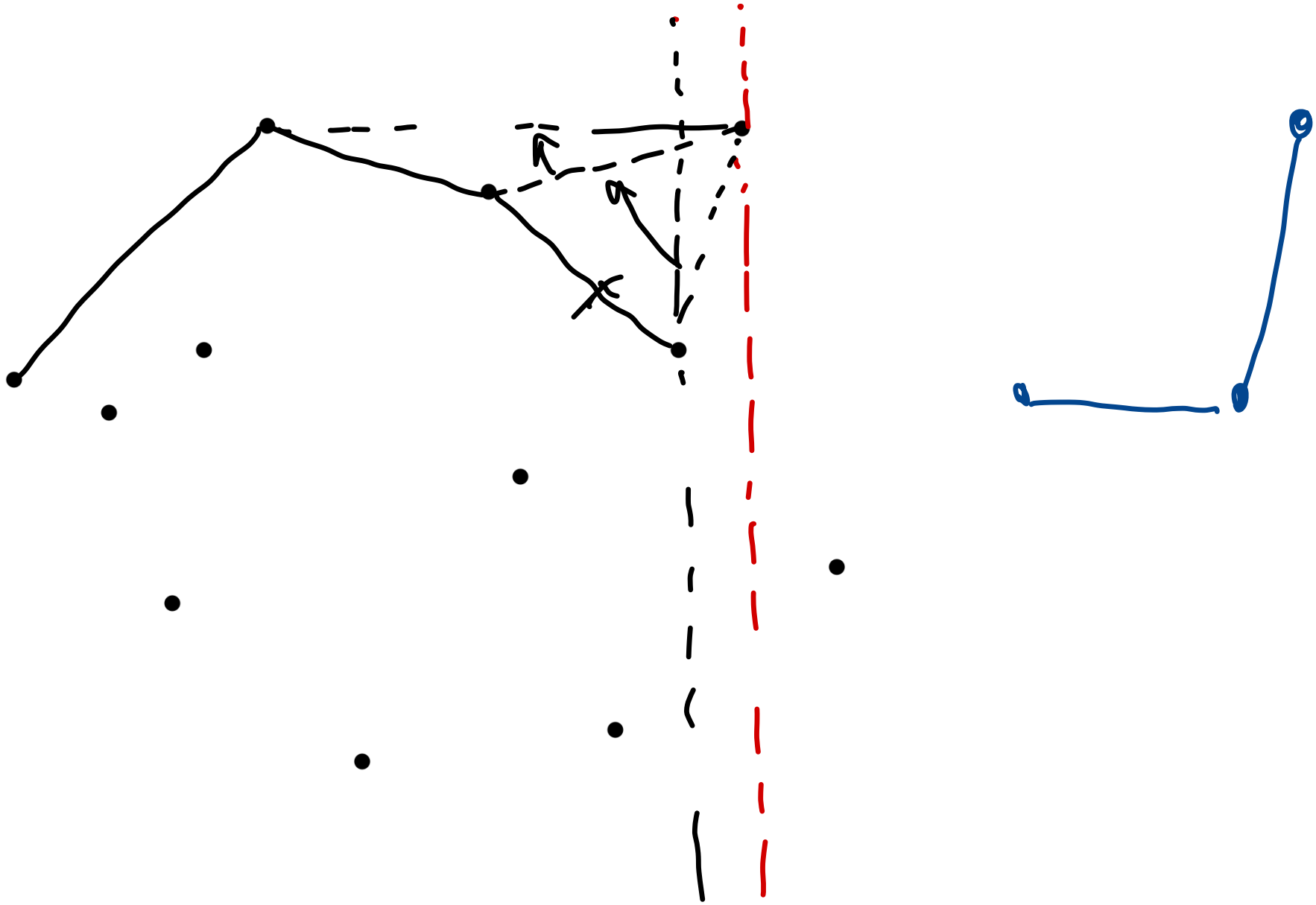
When Graham's Scan completes, `stk` stores the points along the upper boundary of the convex hull of X.

Why?

Must show:

1. Sequence of points in `stk` make only right turns. ← condition of while loop
2. All points in X are below path formed by points in `stk`

Popping Moves Boundary Up



Graham's Scan Efficiency?

Brute force: $\Theta(n^3)$

If there are n points, what is the running time of Graham's scan?

- assume: stack operations are $O(1)$, "left turn" is $O(1)$

Obs. Each point is only push/popped at most once

is left turn

$\Rightarrow$ total # of stack ops $= O(n)$

Also each compare results in 1 stack op, so # compares = # stack ops

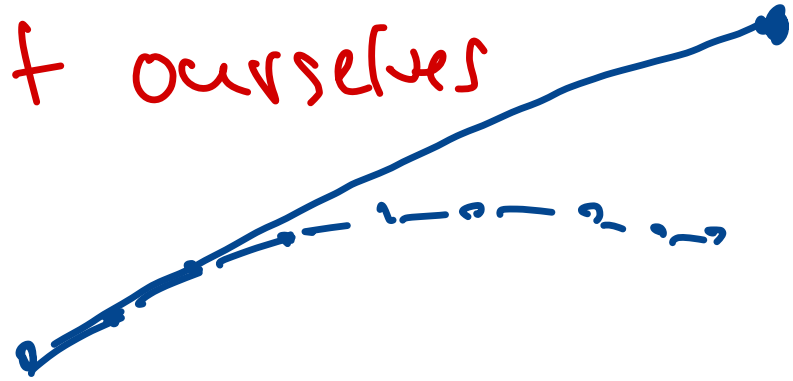
$c \cdot f = O(f)$

$f + g = O(\max(f, g))$

$f * g = O(f * g)$

If we need to sort ourselves
→ $\Theta(n \log n)$

Pseudocode Again



- X sorted by x-coordinate
- `stk` a stack, initially storing first two points in X

For each remaining C in X:

- if `stk.size() == 1`, `stk.push(C)`
- otherwise
  - A and B are top two elements in `stk`
  - while ABC is not a right turn and `stk.size() > 1`
    - `stk.pop()`, update A, B
  - `stk.push(C)`

$\Theta(n)$ iteration

$\Theta(n)$ iteration

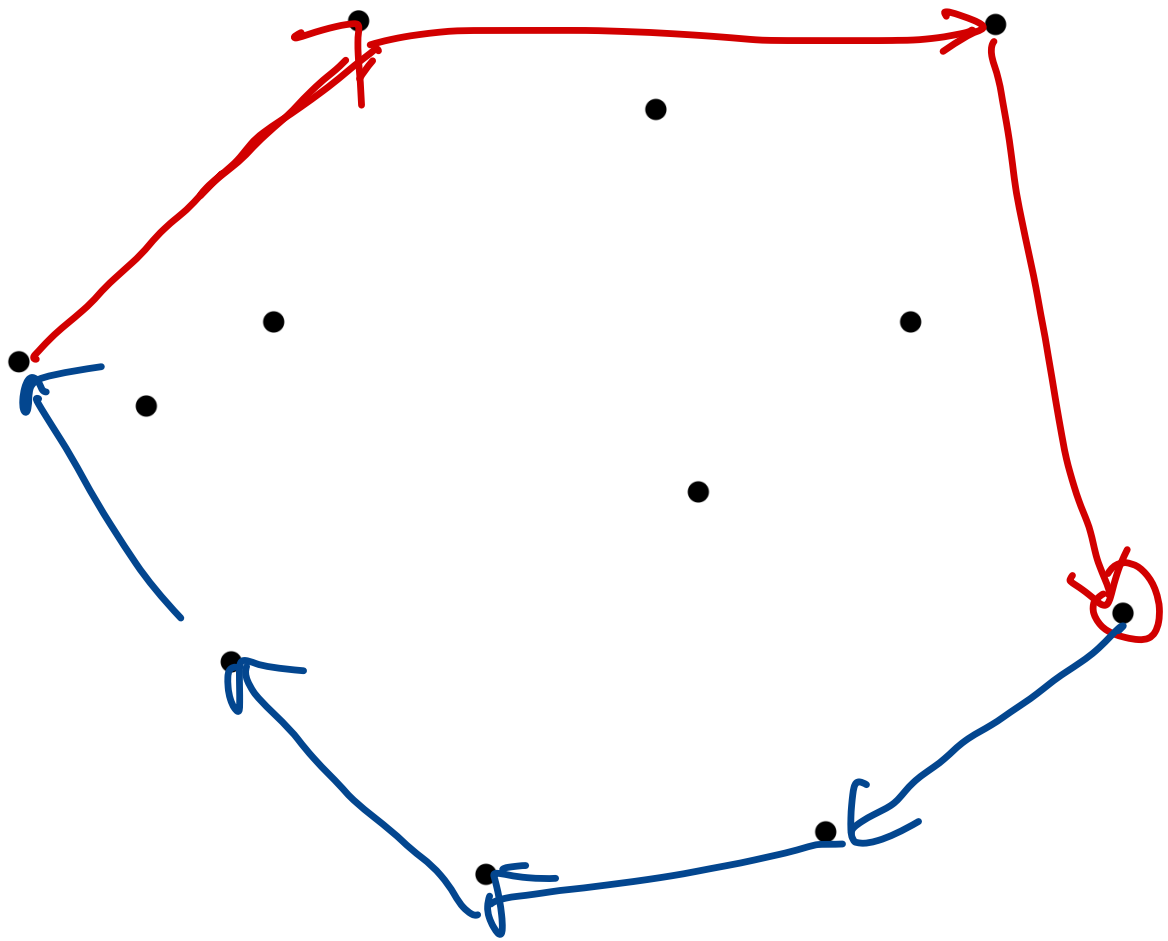
$\Theta(n * n) = \Theta(n^2)$

Running is $\Theta(n)$ if given sorted X,
$\Theta(n \log n)$ if we must sort.

Finishing the Computation

How to find the **lower** boundary of $CH(X)$?

Lower Boundary Illustrated



Perform precisely same scan alg. but w/ X sorted from right to left

Assignment 06

Make an interactive visualization for Graham's scan algorithm

- user can add points in the plane
- program steps through execution and illustrates each step
- returns convex hull of points
- separate non-interactive method for testing

Complete description coming soon!

Animation

Updated DFS Visualization

1. Now highlights current node.
2. Has an **animate** button

Demo!

- `lec10-dfs-animated.zip`

Highlighted Vertex

`GraphVisualizer` changes

1. Added new SVG layer: `overlayGroup`
   - `<g></g>` element for "group"
   - sits "above" other layers (edges, vertices)
2. Methods:
   - `addOverlayVertex(vtx)`
   - `moveOverlayVertex(vtx1, vtx2)`
   - `removeOverlayVertex(vtx)`

`Dfs` changes

- create and move overlay vertex for `cur` vertex

A Simple Goal

Goal 1. Show motion of highlighted vertex

- highlight doesn't "jump" from one frame to the next

Simple Animation with CSS

For this animation:

- only changes are to two attributes of a `circle` object
  - `cx` and `cy` change
- one-shot animation in response to change

Such things can be animated with CSS!

- have class `overlay-vertex`

CSS Transitions

```
/* list the properties to apply transition to */
transition-property: cx, cy;

/* how long should transition last?*/
transition-duration: 500ms;

/* how long
/* some options: ease (default), ease-in, ease-out, ease-in-out,
transition-timing-function: ease;

/* should we delay the start of the transition? */
transition-delay: 500ms;
```

Demo Animation!

A Less Simple Goal

Goal 2. Animate an entire execution of DFS without manually stepping through.

- How fast should steps be?

method(a, b) every 1000ms

setInterval(method, 1000, a, b)

JavaScript Timed Iteration

`setInterval(method, time, arg1, ...)` method:

- call `method` with arguments `arg1, ...` every `time` ms, until eternity
- returns the ID of an `Interval`
- to stop, use `clearInterval(id)` method

The `method` is called a **callback method**

DFS Animation

Repeatedly call `step()` function until algorithm terminates

I used three methods:

- `animate` to start the animation
- `animateStep` to decide what to do for a single animation step
  - call `step()` if the algorithm isn't done
  - stop the animation if it is done
- `stopAnimation` to stop the animation

Starting the Animation

```
this.animate = function () {
    if (this.curAnimation == null) {
        this.start();
        this.curAnimation = setInterval(() => {
            this.animateStep();
        }, 1000);
    }
}
```

check no ongoing animation

~~setInterval(this.animateStep, 1000)~~

Animating a Step

```
this.animateStep = function () {
    if (this.active.length > 0) {
        console.log("taking a step from vertex "
        + this.cur.id);
        this.step();
    } else {
        this.stopAnimation();
    }
}
```

check if alg has not terminated

Stopping the Animation

```
this.stopAnimation = function () {
    clearInterval(this.curAnimation);
    this.curAnimation = null;
    console.log("animation completed");
}
```

Demo Animation

Animation: A Third Way

- `lec10-bouncing-ball.zip`

Goal. Animate a bunch of dots that bounce around the screen!

- simple movement
- indefinite animation
  - no fixed start/end position

Bouncing Ball Demo

Bouncing Ball Implementation

`window.requestAnimationFrame(callback)`:

- perform operations **one time**, then redraw screen
  - precise timing is set by system
- to animate motion, must call `requestAnimationFrame` for each frame
  - this is typically done by having `callback` recursively call the function making the request

Dot Animation Example

Each `dot` executes this code:

```
this.animate = function () {
    this.updateLocation(this.cx + this.vx, this.cy + this.vy)
    if(this.cx <= 0 || this.cx >= WIDTH) {
        this.vx = -this.vx;
    }
    if(this.cy <= 0 || this.cy >= HEIGHT) {
        this.vy = -this.vy;
    }
    window.requestAnimationFrame(() => this.animate());
}
```

Have a Nice Break!