Homework 2

COSC 373, Spring 2022

Will Rosenbaum
Updated: March 2, 2022

Department of Computer Science
Amherst College

Question 1. The problem `MCM` is to find a *maximum cardinality matching* in a network—i.e., a maximal matching in which the most possible vertices are matched. Use an indistinguishability argument to show that `MCM` requires at least $D/2$ rounds in (some) graphs with diameter D. Specifically, you should find two graphs G_1 and G_2 containing vertices v_1 and v_2, respectively, such that (1) v_1 and v_2 have identical distance $D/2-1$ neighborhoods, and (2) v_1 and v_2 must have different partners in maximum cardinality matchings in G_1 and G_2. *(Hint: what is the simplest graph you can imagine with diameter D?)*

Question 2. Recall that the LOCAL model is the synchronous message passing model in which all nodes have unique IDs, and there are no restrictions on message size. Suppose $G = (V, E)$ is a network with $n = |V|$ nodes and diameter D.

(a) For any natural number d, consider the problem d-`NEIGHBORHOOD` which requires each node v to output its distance d neighborhood, $\Gamma_d(v)$. Describe an algorithm in the LOCAL model that solves d-`NEIGHBORHOOD` in $d+1$ rounds. (Hint: you may find the Neighborhood Covering Lemma from Lecture 05 helpful.)

(b) In the problem `OUTPUT_GRAPH`, each node should output the entire network G. Adapt your algorithm from part (a) to show that `OUTPUT_GRAPH` can be solved in $D + 2$ rounds. Be sure to address the issue of determining when each node should halt.

(c) Use the conclusion of part (b) to argue that *every* graph problem can be solved in $D + 2$ rounds in the LOCAL model.

(d) Assuming that the network has n vertices and m edges, and that the ID of each node in the network is encoded with $O(\log n)$ bits, how large (i.e., number of bits) are the largest messages sent by your protocol from part (b)?

Question 3. Recall that the CONGEST model is identical to the LOCAL model, except that nodes can only send messages of size $O(\log n)$ bits, where n is number of nodes in the network. The graph problem `SIZE` requires each node in the network to output n.

(a) Use an indistinguishability argument to show that `SIZE` cannot be solved in fewer than D rounds on graphs with diameter D.

(b) Devise an algorithm *in the CONGEST model* that solves `SIZE` in $O(D)$ rounds in any graph with diameter D. *(Hint: start by using the procedure described in class to perform leader election/BFS tree construction.)*